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New Fractional Order Chaotic System: Analysis, Synchronization, and it's Application

Zain-Aldeen S. A. Rahman[1,2], Basil Hani Jasim*[,2], Yasir I. A. Al-Yasir[3]
[1]Dept. of Electrical Techniques Technical Institute / Qurna, Southern Technical University
Basra, Iraq
[2]Electrical Engineering Department, College of Engineering, University of Basrah, Basra, Iraq
[3]School of Electrical Engineering and Computer Science, Faculty of Engineering and Informatics, University of Bradford, Bradford BD7 1DP, UK

Correspondence
*Basil Hani Jassim
Electrical Engineering Department,
University of Basrah, Basrah, Iraq
Email: hanbas632@gmail.com

Abstract
In this paper, a new nonlinear dynamic system, new three-dimensional fractional order complex chaotic system, is presented. This new system can display hidden chaotic attractors or self-excited chaotic attractors. The Dynamic behaviors of this system have been considered analytically and numerically. Different means including the equilibria, chaotic attractor phase portraits, the Lyapunov exponent, and the bifurcation diagrams are investigated to show the chaos behavior in this new system. Also, a synchronization technique between two identical new systems has been developed in master- slave configuration. The two identical systems are synchronized quickly. Furthermore, the master-slave synchronization is applied in secure communication scheme based on chaotic masking technique. In the application, it is noted that the message is encrypted and transmitted with high security in the transmitter side, in the other hand the original message has been discovered with high accuracy in the receiver side. The corresponding numerical simulation results proved the efficacy and practicability of the developed synchronization technique and its application

KEYWORDS: Attractor, Chaotic, Fractional order, Synchronization, System.

I. Introduction

Chaotic complex systems have a great practice significance research where chaos theory was first presented by Lorenz in 1963 [1]. The recent studies, classify the chaotic attractors int two main classes: self-excited attractors and hidden attractors [2]. The unstable equilibrium point(s) of the nonlinear dynamic system is responsible for exciting the attraction basin in the self-excited chaotic attractor [3]. In the other hand, the hidden attractor can be excited from a dynamical system with stable equilibrium point(s), without equilibrium points, and a line or surface of equilibrium points [4]. In the recent years, chaotic complex systems with hidden attractors have established significant attention [5]. Hidden attractors are very vital in the many fields of since and engineering, like a bridge wings design [6], induction motors for drilling systems [7], aircraft control systems [8], PLL circuits [9], and secure communication schemes [10]. Since Sprott designed chaotic systems in 1994, designing unique chaotic systems remained active field in nonlinear dynamics systems[11] . Many scholars have been constructed chaotic systems with the different and specific dynamics as in [12][13][14][15][16][17].

The earlier hidden attractors studies mostly focused on the integer order dynamical system. Many researches on complex chaotic systems with hidden chaotic attractors were considered such as CAMO: self-excited and hidden chaotic flows[18], hidden attractors with incommensurate fractional order system[19], new examples of hidden chaotic attractors[20], and hidden chaotic attractors excited from chaotic maps[21]. Recently, the fractional order derivative and fraction order integration calculus has established much consideration, that is because of the fractional calculus providing more truthful models other than the integer order [22]. The fractional order chaotic models display even more complex dynamical behavior over the analogues integer models (it's contain the fractional order parameter as well as the original system parameters); thus, they play a significant role in the secure communications systems [23].

The control synchronization is the main issue of using fractional-order chaotic systems in secure communication applications and cryptography [24][25]. Many control systems have been introduced to control and synchronize of fractional-order chaotic systems such as sliding mode control [26], active control [27], adaptive control[28]. impulsive control[29] , and passive control [30]. Many fractional order chaotic systems have been investigated and used insecure communication schemes as in [31][32][33][34].

In this work, we proposed a new 3D fractional order chaotic system, where this system can excite hidden chaotic attractors or self -excited chaotic attractor based on its fractional order that will be used. The dynamical behavior properties of the proposed system are investigated in details in this paper. Then, tow identical new fractional order system has been synchronized based on adaptive control. These two identical systems are the master (derive) and the slave (response). Based on Lyapunov theory the adaptive control law was designed for deriving the slave system. In the same time the update control laws have been determined. This parameter update law is responsible for estimating the uncertain slave parameter corresponding the master parameters. The developed synchronization has been applied in secure communication based on chaotic masking technique, where the master and slave act the transmitter and receiver respectively in the communication system. The results have been obtained by using MATLAB.

The rest of this paper is organized as follows. In section II, the fundamental calculus of the fractional order systems has been introduced. A new 3-D fractional order system with stable equilibrium point(s) or a line of equilibrium is described in section III. In section IV, the dynamical behavior properties of the proposed system are studied involving Lyapunov exponents, and the bifurcation diagrams. In Section V, we introduced a synchronization between two identical systems. In section VI, the we applied the synchronization process in secure communication scheme based on chaotic masking technique. In Section VII, we conclude this paper.

II. PRELIMINARIES

Fractional calculus is a fundamental branch of mathematics, first hypothesized since1695 in a series of letters [35]. There are different definitions of the fractional order derivatives such as Caputo definitions, Gr¨unwald-Letnikov, and Riemann-Liouville. Caputo derivative is a time domain computation method. Caputo derivative is the most general used in real applications [36]. With fractional order (q -order) of a function f (t), the Caput derivative is stated as following [37]:

$$ {}_{t_0}D_t^q f(t) = \begin{cases} \frac{1}{\Gamma(m-q)} \int_{t_0}^{t} \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau ; & m-1 < q < n \\ \frac{d^m f(t)}{dt^m} ; & q = m \end{cases} \quad (1) $$

Where m is the smallest integer number, larger than q and $\Gamma(m-q)$ is defined as Gamma function is the most function used in the fractional calculus, and it's given by [38]:

$$ \Gamma(X) = \int_0^{+\infty} e^{-t} t^{x-1}\, dt \quad ; \quad x > 0 \quad (2) $$
$$ \Gamma(1) = 1; \quad \Gamma(0) = +\infty $$

III. A NEW FRACTIONAL ORDER CHAOTIC MODEL DESCRIPTION

The fractional order systems have been classified into two classes commensurate fractional order and incommensurate fractional order. Int the first type the fractional orders of the system equations are equaled i.e. ($q_1=q_2=\ldots=q_n$), while ($q_1\neq q_2\neq\ldots\neq q_n$) in the second class [39]. In this work the proposed system is a commensurate order system. The mathematical model of new three-dimensional fractional order chaotic system offered in this paper can be described by the following state equations:

$$ \begin{aligned} \frac{d^q x}{dt^q} &= xy - af(z) \\ \frac{d^q y}{dt^q} &= b - x^2 \\ \frac{d^q z}{dt^q} &= x - cz \end{aligned} \quad (3) $$

Where x, y, and z are the state variables; a, b, and c are the positive constant system parameter, q is the fractional order (0<q<1), and sgn(z) is the signum function, which is described as:

$$ f(z) = \begin{cases} 1 & \text{for } z > 1 \\ 0 & \text{for } z = 0 \\ -1 & \text{for } z < 1 \end{cases} \quad (4) $$

The system (3) shows the chaotic behavior for not small range of its parameters values a, b, and c, and the fractional order (q). For the numerical simulation the system parameters are chosen as a=2, b =5, c=1.5, and q = 0.98 with initial conditions (x_0, y_0, z_0) = (0.1,0.1,0.1). The corresponding time series of the system states and system phase portraits as its projections on the different planes are obtained as shown in Fig. 1 and Fig. 2 respectively.

The equilibrium points (equilibria) of the new proposed fractional order chaotic system (3) can be obtained by solving the following nonlinear equations:

$$ \begin{aligned} \frac{d^q x}{dt^q} &= xy - af(z) = 0 \\ \frac{d^q y}{dt^q} &= b - x^2 = 0 \\ \frac{d^q z}{dt^q} &= x - cz = 0 \end{aligned} \quad (5) $$

With chosen parameters (a=2, b =5, c=1.5), the system (5) has solution as in Table 1, that give equilibria of system (3).

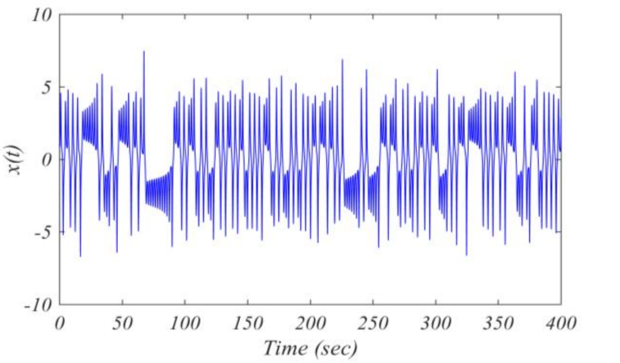
By linearizing, the Jacobian matrix for system (3) have been gotten as:

$$ J = \begin{bmatrix} y & x & 0 \\ -2x & 0 & 0 \\ 1 & 0 & -c \end{bmatrix} \quad (6) $$

In the fractional order systems, the equilibrium point is stable if it's satisfied the following condition:

$$ |arg(\lambda_i)| > q\frac{\pi}{2} ; \; i=1,2,3 \quad (7) $$

According to that condition, the chaotic attractor of this system (3) depend on the fractional order value (q), it's may by a self-excited chaotic attractor or a hidden attractor.



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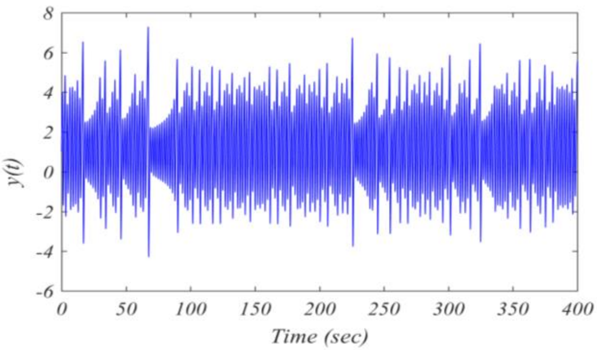

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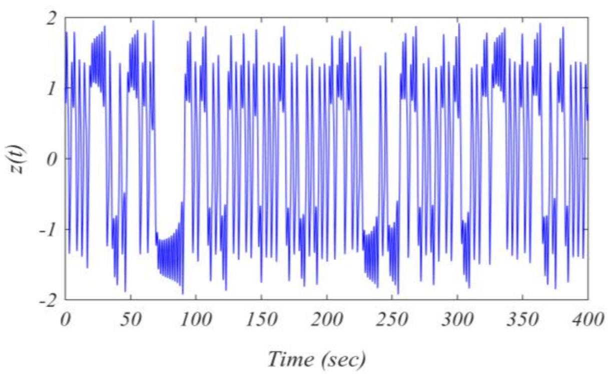


(c)

Fig. 1. The proposed system states time seies; (a) x(t), (b) y(t), (c) z(t)

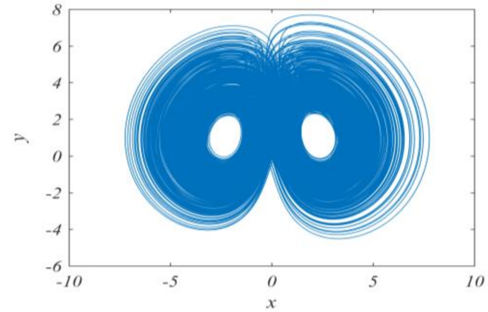
TABLE 1
THE EQUILIBRIA AND CORRESPONDING EIGENVALUES OF SYSTEM (3).

f(z)	E(x*,y*,z*)	Eigenvalues
-1	E_1(2.23, -0.894, 1.5)	λ_1= -1.5, $\lambda_{2,3}$= -0.4472± 3.1305i
	E_2(-2.23, 0.894, -1.5)	λ_1= -1.5, $\lambda_{2,3}$=0.4472± 3.1305i
1	E_1(2.23, 0.894, 1.5)	λ_1= -1.5, $\lambda_{2,3}$=0.4472± 3.1305i
	E_2(-2.23,-0.894, -1.5)	λ_1= -1.5, $\lambda_{2,3}$= -0.4472± 3.1305i
0	E_1(2.23,0, 1.5)	λ_1= -1.5, $\lambda_{2,3}$= ± 3.1536i
	E_1(-2.23,0, -1.5)	λ_1= 1.5, $\lambda_{2,3}$= ± 3.1536i

Since q = 0.98 is selected, that satisfied condition (7), thus the equilibrium points are classified as the following:

Case 1: When f(z)= ±1; the fixed points $E_{1,2}$ does not satisfy the condition (7), thus these fixed points are unstable. Therefore, the system (3) exhibits self-excited chaotic attractors.

Case 2: When f(z)=0; the fixed pint E_1(2.23,0, 1.5) satisfy the condition (7), thus this fixed point is a stable. Therefore, the system (3) can excite hidden chaotic attractors.



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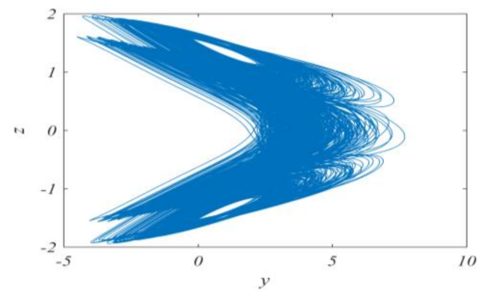

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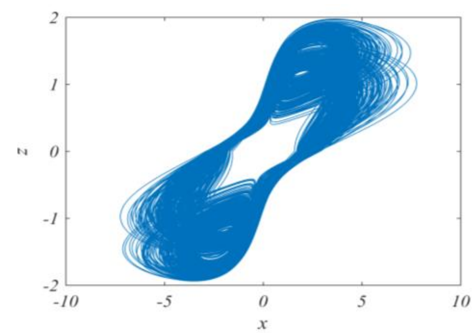

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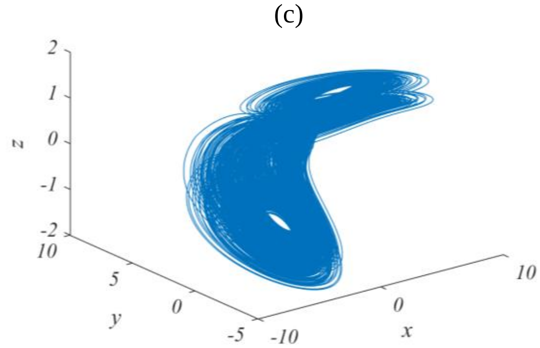

(d)

Fig. 2. The chaotic attractor of system (3); (a) x-y phase plane, (b) y-z phase plane, (c) x-z phase plane, (d) three-dimensional view.

IV. A System Dynamical Behavior

In this section, basic dynamical properties of system (3) are numerically investigated by Lyapunov exponents and bifurcation diagrams using MATLAB platform.

A. *Lyapunov Exponents*

For improving the proposed system exhibits a chaotic attractor, the Lyapunov exponents have been determined. In Fig. 3, The Lyapunov exponents are determined with respect to time (1000 sec). The selected system parameters are (a=2,b=5, and c=1.5), and the fractional order (q=0.98) with initial condition as (x_0, y_0, z_0) = (0.1,0.1,0.1), the corresponding determined Lyapunov exponents are Le1 = 0.5022, Le2 = −1.4823, and Le3 = −1.6595. There is a positive Lyapunov exponent (Le1), thus proposed system (3) exhibits chaotic behavior. In addition, since $Le1 + Le2 + Le3 = -2.6396 < 0$, it indicates that system (3) is dissipative (i.e., the system state trajectories ultimately settle down into a strange attractor). The Lyapunov exponents are calculated numerical by MATLB platform according to[40].

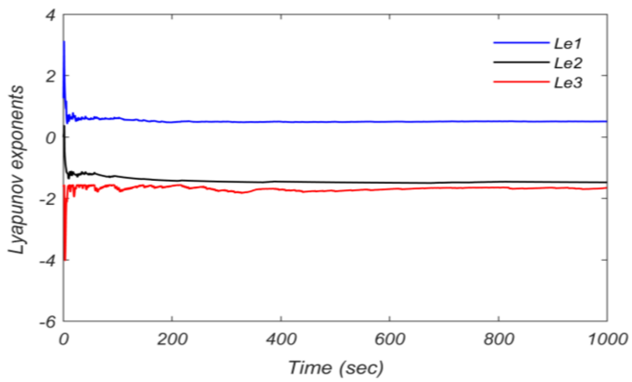

Fig. 3. Lyapunov exponents with respect to time

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Furthermore, for the parameter values (a=2, 5, b=5, and c=01.5) and initial values (x_0, y_0, z_0) = (0.1,0.1,0.1), the Lyapunov exponents are determined verse changing the fractional order q ∈ [0.85,1] as in Fig. 4. In Fig. 4, The Lyapunov exponents are obtained as; Le1 = 0.4803, Le2 =0.0804, and Le3 = −1.5035. That's strongly indicates the proposed system exhibits chaotic attractors.

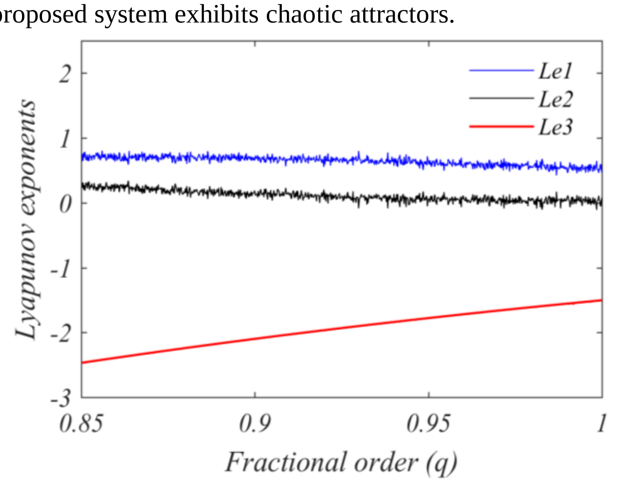


Fig. 4. Lyapunov exponents with respect to fractional order (q)

B. *Bifurcation Diagrams*

The bifurcation diagram is a significant tool in the nonlinear theory and chaos. In this worked, the bifurcation diagrams of the state variable x against the parameter d also, with respect to the fractional order (q) of the system have been introduced to show the dynamical behavior of the new system (3).

By setting the parameters b = 5, c= 1.5, the initial conditions (0.1, 0.1, 0.1), the fractional order (q=0.98), and selecting a to be the bifurcation parameter. The influence of parameter a on the system (3) dynamical behavior have been determined by the bifurcation diagram as illustrated in Fig. 5. As it's noted in Fig. 5, the new system (3) can excite the chaoticity phenomena when parameter a >1.2.

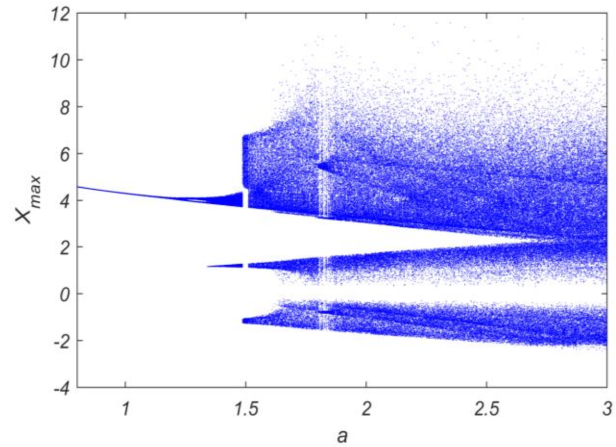


Fig. 5. The Bifurcation diagram with variation of system parameter (a)

Then, by setting the parameters a = 2, b = 5, c = 1.5, the initial conditions (0.1, 0.1, 0.1), and choosing the fractional order (q) to be the bifurcation parameter. Fig. 6, displays the state variable x verses the changing fractional order in a form of bifurcation diagram. As inferred in Fig. 6, that chaos behavior may exist for the fractional order q > 0.895.

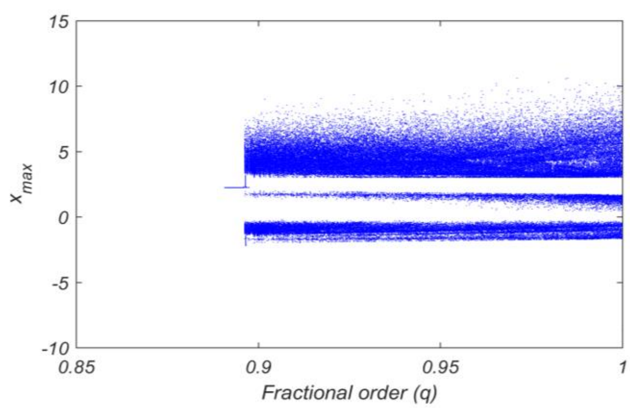


Fig. 6. The Bifurcation diagram with variation of fractional order (q)

V. Synchronization Strategy

In this part, a synchronization technique between two new identical systems has been realized, one present the derive (master) and other present the controlled (slave). The synchronization scheme has been verified so that the slave trajectories asymptotically match the corresponding master trajectories. The synchronization of the fractional order chaotic systems has a significant potential in a secure communication field application. With the growth of the research, there are various synchronization techniques have been proposed [41]. In this paper, an adaptive controller is designed for realizing the system synchronization control.

A. Synchronization Controller Development

For realizing the synchronization, consider the master and slave systems are given as in (8) and (9) respectively.

$$\begin{aligned} \frac{d^q x_m}{dt^q} &= x_m y_m - a_m f(z_m) \\ \frac{d^q y_m}{dt^q} &= b_m - {x_m}^2 \\ \frac{d^q y_m}{dt^q} &= x_m - c_m z_m \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{d^q x_s}{dt^q} &= x_s y_s - a_s f(z_s) + u_1 \\ \frac{d^q y_s}{dt^q} &= b_s(t) - {x_s}^2 + u_2 \\ \frac{d^q y_s}{dt^q} &= x_s - c_s(t) z_s + u_3 \end{aligned} \tag{9}$$

Where, u_1, u_2, and u_3 are the adaptive controllers want to be designed, $b_s(t)$ and $c_s(t)$, are the uncertain slave parameters will be estimated. The master-slave synchronization errors can be calculated as following:

$$\begin{aligned} e_x &= x_s - x_m \\ e_y &= y_s - y_m \\ e_z &= z_s - z_m \end{aligned} \tag{10}$$

Then, the obtained corresponding error dynamics:

$$\begin{aligned} \frac{d^q e_x}{dt^q} &= x_s y_s - a_s f(z_s) - x_m y_m + a_m f(z_m) + u_1 \\ \frac{d^q e_y}{dt^q} &= e_b - {x_s}^2 + {x_m}^2 + u_2 \\ \frac{d^q e_z}{dt^q} &= e_x - c_s(t) e_z + e_c z_m + u_3 \end{aligned} \tag{11}$$

Where e_b and e_c, are the error of master-slave parameter estimation and defined by the following equations:

$$e_b = b_s(t) - b_m \; ; \; e_c = c_s(t) - c_m \tag{12}$$

That results:

$$\dot{e}_b = \dot{b}_s(t) \; ; \; \dot{e}_c = \dot{c}_s(t) \tag{13}$$

Now consider the quadratic Lyapunov function as:

$$V(e_x, e_y, e_z, e_b, e_c) = \frac{1}{2}(e_x^2 + e_y^2 + e_z^2 + e_b^2 + e_c^2) \tag{14}$$

By applying the fractional order time derivative for (14), we get:

$$\dot{V} = (e_x \frac{d^q e_x}{dt^q} + e_y \frac{d^q e_y}{dt^q} + e_z \frac{d^q e_z}{dt^q} + e_b \dot{e}_b + e_c \dot{e}_c) \tag{15}$$

The following equation is obtained by substituting equations (11-13) in (15).

$$\begin{aligned} \dot{V} = (&e_x(x_s y_s - a_s f(z_s) - x_m y_m + a_m f(z_m) + u_1) + \\ &e_y(-{x_s}^2 + {x_m}^2 + u_2) + e_z(e_x - c_s(t) e_z + u_3) + \\ &e_b(\dot{e}_b + e_y) + e_c(\dot{c}_s(t) + e_c z_m) \end{aligned} \tag{16}$$

The adaptive controllers have been designed as:

$$\begin{aligned} u_1 &= -k_x e_x - x_s y_s + a_s f(z_s) + x_m y_m - a_m f(z_m) \\ u_2 &= -k_y e_y + {x_s}^2 - {x_m}^2 \\ u_3 &= -k_z e_z - e_x + c_s(t) e_z \end{aligned} \tag{17}$$

Where, k_x, k_y, and k_z, are positive constants. Additionally, the update law of the uncertain slave parameter can be selected:

$$\dot{b}_s(t) = -e_y; \; \dot{c}_s(t) = -z_m e_z \tag{18}$$

Finally, by substituting the adaptive controllers (17) and update low (18) in (16) we get.

$$\dot{V} = -k_x {e_x}^2 - k_y {e_y}^2 - k_z y^2 < 0 \tag{19}$$

Hence, it is immediate that e(t) converges exponentially to zero with respect to time for all initial conditions. Equation (19), guarantees the asymptotic global stability of developed synchronization technique.

B. Simulation Results

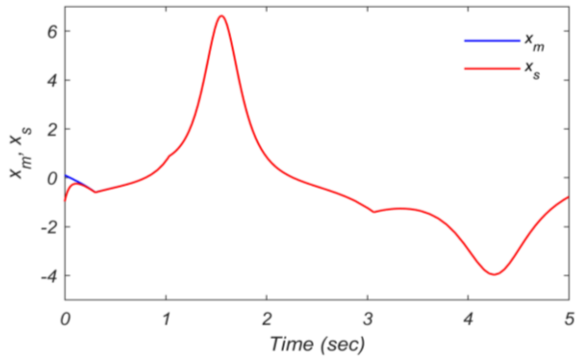
In order to confirm the effectiveness of the suggested synchronization technique, numerical experiments are applied by using MATLAB platform. The synchronization results are obtained with fractional order (q=0.98), the master system (5) parameters are (a_m=2, 5, and b_m=5, c_m=1.5),), the slave system (6) parameters are (a_s=2, b_m=uncertain, and c_s=uncertain), and initial conditions for the master and slave are (0.1, 0.1, 0.1) and (−1, −0.5, 1) respectively.

By using the derived adaptive control law (17), and the slave parameters update law (18), the slave trajectories follow the master trajectories effectively as illustrated in Fig. 7.

The time history of the synchronization errors e_x, e_y, and e_z are displayed in Fig. 8. From Fig. 8, it's clear that the large synchronization errors are converge to zero rapidly (in less than 1.7 seconds). The simulation results show that the master and slave state variables have been synchronized in a small time although the initial conditions have different signs and different values therefore, the designed controller is effective. According to the update law (18), these uncertain slave parameters have been estimated correctly to the corresponding master parameters as shown in Fig. 9.

VI. Secure Communication Application

This section, introduces the suitability of the fractional order chaotic systems for application in secure communication schemes. The fractional order chaotic signal is characterized by its extreme sensitivity to the initial condition and the system parameters, the random like nature and broadband spectrum[42] .The broadband nature is the main reason for increasing the security of communication based on the fractional order chaotic signals[43].



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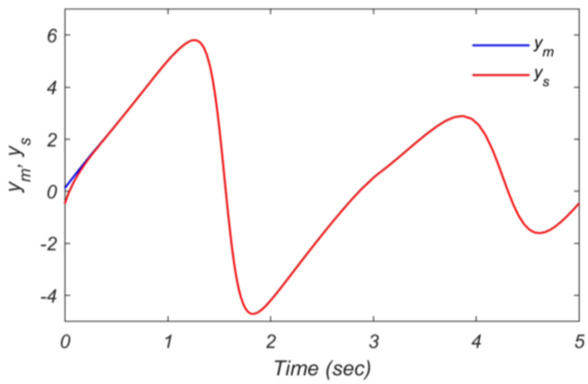

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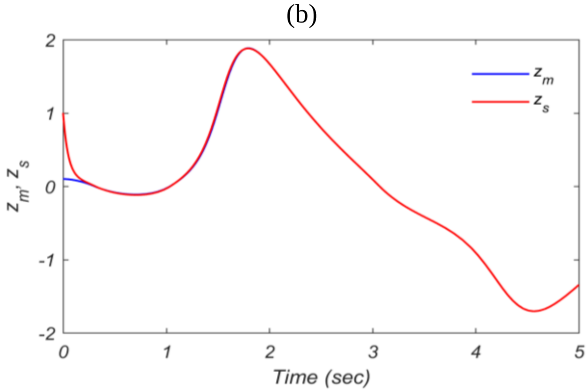


(c)

Fig. 7. Master-Slave states synchronization; (a) x_m-x_s , (b) y_m-y_s, (c) z_m-z_s

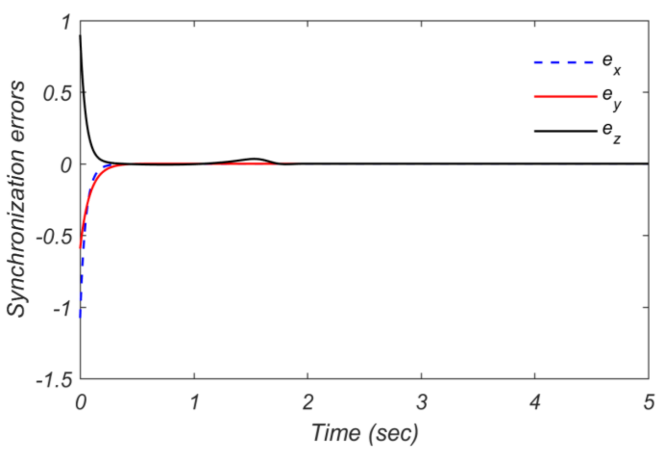
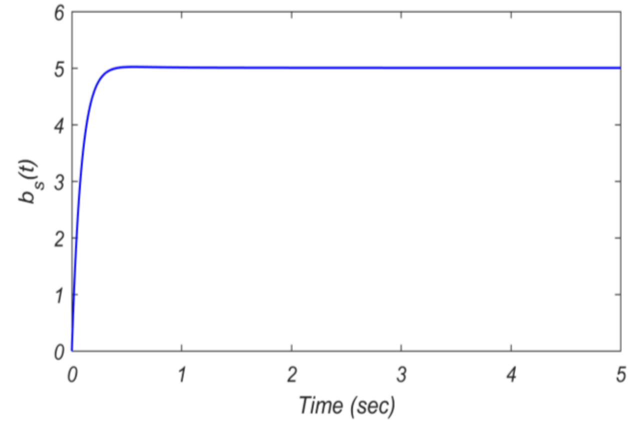


Fig. 8. Master-Slave states synchronization errors



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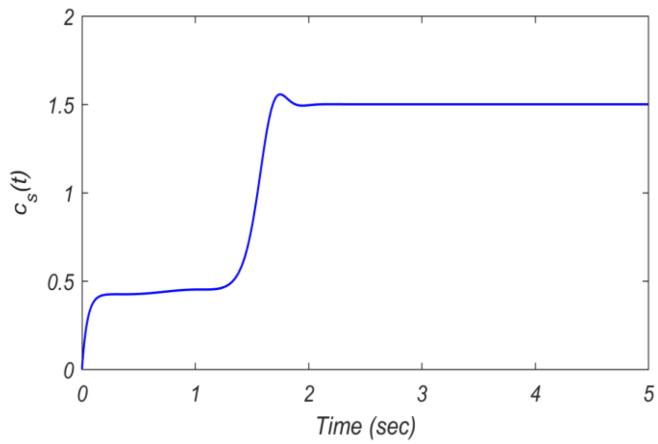


(b)

Fig. 9. The uncertain slave perimeters estimation; (a) parameter b_s, (b) parameter c_s.

There are many techniques for using chaotic signal in secure communication application as parameter modulation, inclusion, chaotic masking, and chaotic shift keying[44].

In this paper, the chaotic masking has been used. Chaotic masking technique, includes adding a message signal to a chaotic signal in the transmitter (master) side, resulting encrypted signals that will be transmitted to the corresponding receiver. The block diagram demonstrating the basic of chaotic masking is exposed in Fig. 10.

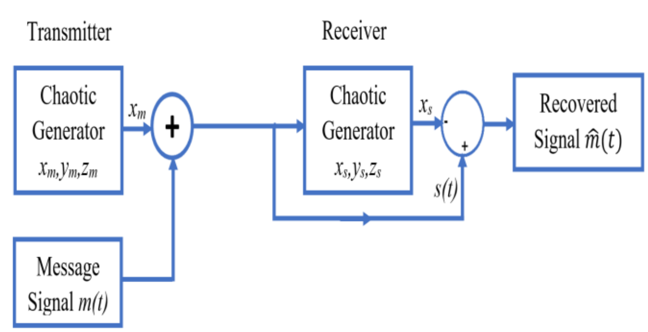


Fig. 10. Communication scheme based on chaotic masking.

Appreciations for chaos synchronization in recovering the original message signals at the receiver side. In other words, the recovery of the original signal involves the receiver must generate a similar copy of the chaotic signals that have been synchronized with the transmitter ones. Then the original message signal is recovered by subtracting the chaotic signal from the total received signal at the receiver side. The amplitude of the message must be very small associated with the chaotic signal, otherwise, the synchronization may be lost and also the chaotic signal will not be able to hide the message spectrum.
Here, the message signal is given by:

$$m(t) = A\sin(2\pi f t) \quad (20)$$

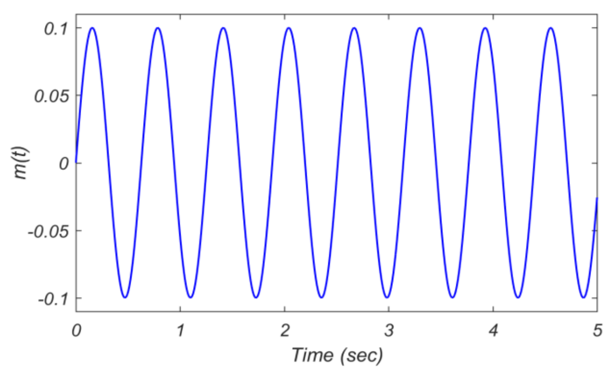
Where A is the amplitude (chosen 0.1) and f is the frequency (chosen 1 Hz) of the message sinusoidal wave signal. The signal (20) is added to the created chaotic $x_m(t)$ signal at the transmitter side, resulting the masked signal to be transmitted as below:

$$s(t)=m(t)+x_m(t) \quad (21)$$

The signal (21), is received at the receiver. In the receiver, the original signal can be retrieved by subtracting the chaotic signal $x_s(t)$ from the masked signal as following:

$$\hat{m}(t) = s(t) - x_s(t) \quad (22)$$

Fig. 11, display the numerical simulation results for the communication scheme based on chaotic masking technique.



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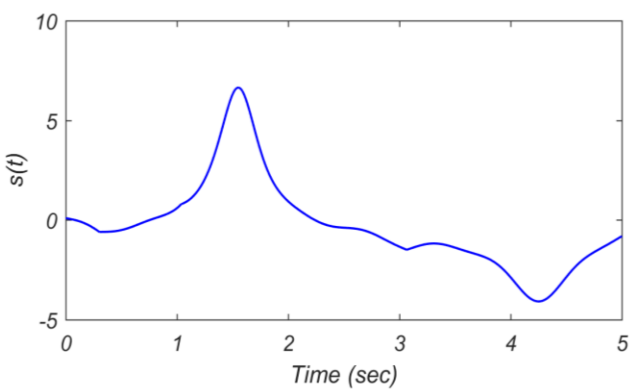
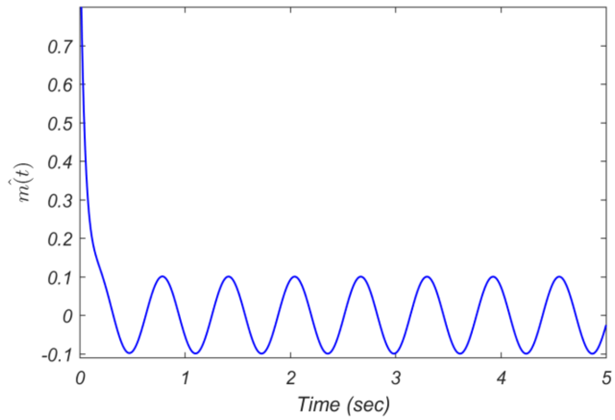


(b)



(c)

Fig 11. Chaotic masking simulation; (a) message signal, (b) chaotic masking transmitted signal, (c) Retrieved signal.

VI. Conclusions

In this article, a new 3D fractional order chaotic complex system has been proposed. The basic dynamical behavior characteristics of the new system have been investigated by different means involving, the equilibrium points and its corresponding eigenvalues, the chaotic attractors, Lyapunov exponents, and the bifurcation diagrams. Since this system possesses stable equilibrium points or a line of equilibrium points, it can exhibit self-excited attractors or hidden attractors. Moreover, a synchronization control of two identical fractional order systems has been verified and its results showed the slave trajectories track and matched the corresponding mastery trajectories quickly. Finally, this synchronization control has been used in secure communication application based on chaotic masking technique. The obtained simulation results showed the good performance for applications in the chaotic masking communication.

Conflict of Interest

The authors have no conflict of relevant interest to this article.

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