

# Chaos Phenomenon in Power Systems: A Review

Abdul-Basset A. Al-Hussein\*

Electrical Engineering Department, University of Basrah, Basrah, Iraq

## Correspondence

\* Abdul-Basset A. Al-Hussein  
Electrical Engineering Department,  
University of Basrah, Basrah, Iraq  
Email: abdulbasset.jasim@uobasrah.edu.iq

## Abstract

*This review article puts forward the phenomena of chaotic oscillation in electrical power systems. The aim is to present some short summaries written by distinguished researchers in the field of chaotic oscillation in power systems. The reviewed papers are classified according to the phenomena that cause the chaotic oscillations in electrical power systems. Modern electrical power systems are evolving day by day from small networks toward large-scale grids. Electrical power systems are constituted of multiple inter-linked together elements, such as synchronous generators, transformers, transmission lines, linear and nonlinear loads, and many other devices. Most of these components are inherently nonlinear in nature rendering the whole electrical power system as a complex nonlinear network. Nonlinear systems can evolve very complex dynamics such as static and dynamic bifurcations and may also behave chaotically. Chaos in electrical power systems is very unwanted as it can drive system bus voltage to instability and can lead to voltage collapse and ultimately cause a general blackout.*

**KEYWORDS:** Bifurcation, Chaos, Power system oscillation, Voltage and angle instability, Blackout.

## NOMENCLATURE

AVR	Automatic voltage regulator.
CB	Cyclic-fold bifurcations.
DOF	Degree of freedom.
$\Delta f$	Frequency difference.
GW	Gigawatts.
HB	Hopf bifurcations
PD	Period-doubling bifurcations.
$P_m$	Generator input mechanical power.
PSD	Power spectral density.
$Q_1$	Reactive power.
SMIB	Single-machine infinite-bus.
SSR	Subsynchronous resonance.
TB	Torus bifurcations.

## I. INTRODUCTION

Electrical power systems are the most sophisticated and important systems in modern society. They have immediate consequences for modernization, politics, economics, and society. Electrical power system service continuity and reliability is a broad research topic, in which power outages and cascade failure analysis play a critical role in creating the future of such systems and ensuring their stable and secure functioning.

Modern electrical power grids are operated close to steady-state stability margin as energy demand develops, which can easily lead to a critical situation. Electric power

systems are made up of a huge number of components that interact in nonlinear ways with each other. These components have a dynamical response that spans a wide variety of time scales. Each time-scale components have their own effects on the dynamical reactions of power systems.

Many research works have been dedicated for enhancing a power system's ability to maintain steady-state and transient stability. The most crucial quantities in power systems to carefully control are voltage, frequency, and rotor angle of synchronous generating units in order to ensure power system stability. The frequency stability is immediately affected by the power imbalance between demand and generation, whereas the voltage is directly affected by the reactive power imbalance [1]. The rotor angle behavior is also indicative of the overall power system's stability and synchronism. If the power system experiences an unusual situation, such as an overload caused by a generator failure, sudden load reconnection, or transmission line tripping, frequency and voltage instabilities must be addressed immediately. If these abnormal conditions are not addressed promptly, the system will undergo cascading events that could result in a blackout [1], [2]. As a result, when a blackout in a power system occurs, the consequences might be far-reaching.

Several blackouts of commercial power systems have occurred, leaving millions of customers stuck for hours without services. For example, on July 31, 2012, a power



This is an open access article under the terms of the Creative Commons Attribution License, which permits use, distribution and reproduction in any medium, provided the original work is properly cited.

© 2021 The Authors. Iraqi Journal for Electrical and Electronic Engineering by College of Engineering, University of Basrah.

outage lasted several hours, affecting roughly 700 million people in India's north and east due to a demand-generation imbalance when nearly 32 GW of electricity was disrupted [3]. This blackout is the greatest power outage ever recorded in terms of the number of persons affected [2].

The behavior of chaos in an electrical power system can manifest as abnormal oscillations under certain situations, posing a threat to the electrical grid's reliability and stability [4]. Chaos research is an important topic due to the nonlinear nature of power grids. This field is until now in its early stages. Where a lot of aspects need to be investigated. In power systems, chaotic occurrences are unavoidable, and it has an essential influence in the system's overall stability notion. [4]. Furthermore, the analysis of the chaos phenomenon opens up new avenues for modern control subjects in electrical power systems, and chaos research is projected to enrich smart grid development in the near future.

This review article is organized as follows. In Section II, some useful definitions relevant to chaotic dynamics are restated and provided firstly. Then, a brief history of the first study that investigated the chaotic behavior in electrical power systems is given in this part. In continuation of the review, Section III discusses the chaos in single-machine infinite-bus (SMIB). Some directions on the effect of hard limit in the synchronous machine excitation system are reviewed in Section IV. Moreover, the noise-induced chaos in power systems, which have been previously reported in the literature, are summarized in Section V. Then the effect of active and reactive power of the load is presented in Section VI. Furthermore, the mechanical power of the generator prime mover is discussed in Section VII. Time delay is unavoidable in power systems, chaos induced in power systems is discussed in Section VIII. In Section IX, some resonance phenomena problems between the electrical and mechanical components of power systems are presented. Then the resonance is revisited but due to the electric circuit of the transmission line and the saturated transformers is demonstrated in Section X. Finally, the review is concluded in Section XI. As per the best of the author's knowledge, this is the first review in this field of chaos in power systems, discuss this phenomenon in such a theme.

## II. PRELIMINARIES DEFINITIONS AND CHARACTERISTICS

### A. Definition of Chaos

Chaos is an aperiodic long-term behavior that occurs in a deterministic system that shows highly sensitive dependence on initial conditions. Chaos is often considered as noise as it has a noise-like appearance, but noise is random in nature whereas chaos is deterministic. The non-linear systems which show chaotic behavior are termed as chaotic systems [5]. Chaotic systems are highly sensitive to the initial conditions as they follow the butterfly effect which tells us that even a slight flapping of wings by a butterfly can cause a hurricane [6]. Chaotic behavior cannot be forecasted for the long term, but it may be predicted in the short term [7].

### B. Characteristic of Chaos

(1) Sensitivity to initial conditions:

Chaos has a high sensitivity to the initial conditions. The initial conditions' small differences will produce vastly diverse outcomes. The butterfly effect is a classic example of small alterations in initial values leading to large disparities.

(2) The randomness nature:

The goal of research in chaos topic is to discover movement laws, which is intrinsically stochastic movement. Chaos is a sort of sequential movement that is far distinct from true random motion. Chaos uncertainty is not generated by external effects, but rather arises spontaneously, and this uncertainty is associated with specific styles.

(3) Positive Lyapunov exponent:

At least one Lyapunov exponent  $\lambda$  in a chaotic system. Should be positive. This existence of this  $\lambda_i > 0$  can be used as a judgment for chaotic behavior in the power system.

(4) The ergodicity:

Chaotic motion is generally constrained within a confined region in phase space and the trajectories are never intersected or regenerated. It is a sophisticated action that differs from normal movements when the concept of certainty is investigated. The chaotic movements can not halt in a specific state over time, instead traversing all points in state space, and the patterns are typically structurally comparable.

(5) The strange attractor:

The attractor in a chaotic state is called a strange attractor which is associated with nonlinear system behavior. The strange attractor indicates the certainty, regularity, and order and in the motion, which is a crucial signal to show the distinction from true stochastic motion. The chaotic attractor, unlike other attractors, has a positive Lyapunov exponent, that can be considered as a key property.

The study of power system chaos began in the early 1980s. N. Kopell, an American academic, studied chaotic movement on a specific surface of energy in swing equations modeling a three-machine power system with two degrees of freedom (2DOF). These models are used in the analysis of the stability of electrical power systems in the transient state [8]. The results demonstrated that the swing equations of a connected power system generate complex dynamics in the form of horseshoe chaos.

## III. CHAOS IN SINGLE MACHINE INFINITE BUS (SMIB)

Fig. 1 depicts a SMIB power system schematic diagram that is considered in this article. The synchronous generator "one" is shown in this diagram, delivering power to the infinite-bus "five" via the main transformer "two", system tie line "three", and impedance "four".

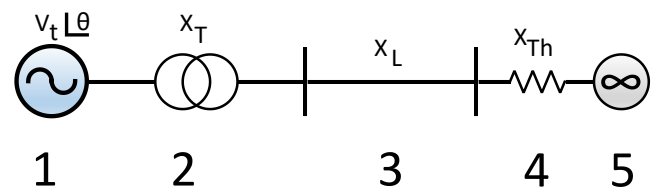


Fig. 1 The configuration of the single-machine infinite-bus (SMIB) power system.

The mathematical model of this system can be described by swing equation and represented as follows:

$$\begin{aligned} \dot{x}_1 &= x_2; \\ \dot{x}_2 &= -cx_2 - \beta \sin x_1 + f \sin \omega t \end{aligned} \quad (1)$$

where

$$x_1 = \theta, x_2 = \dot{\theta}, c = D/M, \beta = P_{max}/M, f = A/M.$$

The authors of [9] in 1983 showed in analytical method, the existence of horseshoe-chaos in the SMIB electrical power system where an excitation system is included on this model. And they used singular perturbation techniques, to reduce the system order, and derive a 4-dimensional nonlinear model, based on the parameters perturbation from a near-integrable 2DOF Hamiltonian system.

The existence of degenerate Hopf bifurcations in an electric power system has been investigated analytically using the Lyapunov-Schmidt theory and through simulation in the work of [10]. Then, to specify the degenerate Hopf bifurcation types, a qualitative method based on system trajectory data is proposed.

One important task in performing bifurcation analysis on nonlinear systems from an engineering perspective, such as in electrical power systems, is investigating the mechanism that leads to the loss of stable equilibrium points owing to a bifurcation, as well as the system dynamical behaviors after the bifurcation. The nonlinear system state will develop in accordance with the system dynamics when a bifurcation occurs. After bifurcation, the dynamics specify if the system will be in a stable state or change to an unstable state, in addition to instability types [11].

In (2000) [12] a basic SMIB electrical power system model, Venkatasubramanian and his coworkers investigated the possibility of the occurrence of Hopf bifurcations. Their primary focus is on Hopf bifurcations nature if subcritical or supercritical when process parameters change. They observed that the subcritical Hopf bifurcations are dominant.

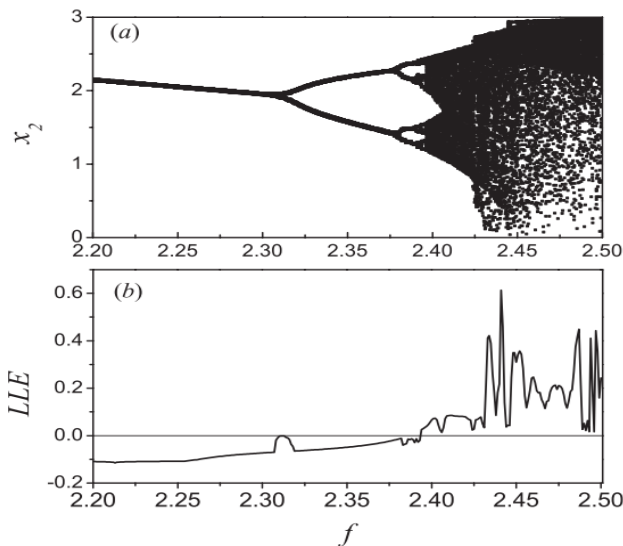


Fig. 2: The bifurcation diagram and the largest Lyapunov exponent with  $f$  as bifurcation parameter. The system parameters are chosen as  $\beta = 1, c = 0.5$ , and  $\omega = 1$ . Adapted from [13].

#### IV. HARD LIMIT INDUCED CHAOS IN POWER SYSTEM

In (1996) [14], a typical model of electrical power system exhibits a cascade of period-doubling bifurcations, which lead to continual chaotic oscillation. Where the gain of the excitation system is made high which is common in industrial applications. The between hard-limits interaction with system transients over a wide range of practical parameter values causes prolonged complex oscillations. A simplified schematic diagram for an excitation system is shown in Fig. 3.

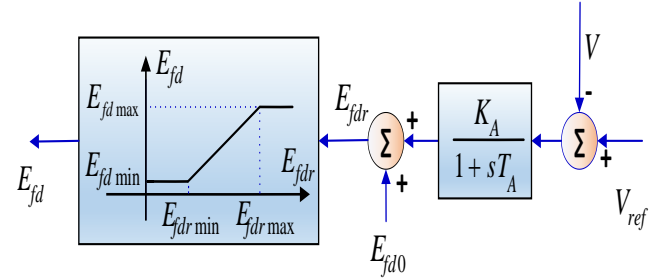


Fig. 3 Feedback excitation system adapted from [15].

In [16] (1999) the existence of a critical phenomenon in nonlinear models is described in this work, where four types of attractors coexist within a basic electrical power system namely a stable equilibrium, a stable limit cycle, and two strange attractors. Even though the model has a feasible stable equilibrium point, the results show that power system operation can become locked in sustained chaotic motion following a large disruption.

The presence of several attractors in a nonlinear dynamical system is critical because it implies that the physical system under examination may have various operating conditions. In industrial applications like the electric power system, stable fixed points are typically the usual operating situations. The existence of a stable limit cycle, a stable equilibrium point, and strange attractors in the same system implies that a power system transient may become stranded in one of the following states:

- Stable fixed point,
- Oscillations state, or
- Chaotic oscillation, totally dependent on the initial condition after disturbance.

It should be noted that the four types of attractors can be feasible practically, as a sustained oscillatory state would be considered transient by the protective relay equipment of the power system [16]. In the real world, protective relays are usually designed to avoid interfering with transient situations. Therefore, in general, the protective equipment cannot act to stop power oscillation due to both stable limit cycles and strange attractors. However, prolonged operation under these circumstances, even for a few minutes, could result in serious fault to important devices like generator rotor shafts. As a result, it's critical to recognize these unwanted oscillations when they arise in the power system, and how the variation of process parameters results in oscillations, for instance as the variation in the output active power of the generator. Since chaotic oscillation generally as defined has a wide frequency spectrum, it can cause

unwanted transients harmonic in generators, in other words, chaotic attractors are a critical problem to normal operation.

## V. NOISE-INDUCED CHAOS IN POWER SYSTEM

In 2009 [13] numerically investigated the effect of Gaussian-white-noise on the dynamical behavior of electrical power networks. The provided scheme is characterized by a SMIB power system and the values of its parameters are chosen such that the system is in a stable state. It has been observed that when the noise level  $\sigma$  increases, the power system shows an unstable state and falls into chaos phenomenon. These findings suggest that random noise can both trigger and intensify chaotic oscillation in the SMIB.

Using the random Melnikov method and numerical simulation, [17] in (2014) investigated how noise-perturbed parameter influences the performance of electrical power systems. The investigated model is formulated as traditional SMIB power systems that work in a stable region far from chaotic nature and all parameters are deterministic. It has been observed in this article that when the perturbations in parameter values are weak, there is no chaos oscillation. As the random variable  $\rho$  is raised in intensity, the dynamical model becomes unstable and then chaotic. These findings suggest that random parameters in power systems can both cause and promote chaos.

In (2010) [18] this work investigates how the noise-perturbed phase (random phase) impacts the dynamical behavior of a basic power system operating in a stable state, and away from chaotic behavior in case of no noise is exist. It is found that when the perturbation in phase is not strong, then there is no chaos in power systems. As the intensity of the disturbance  $\sigma$  grows, power systems become unstable and then chaotic. These results indicate that random phase can both cause and promote power systems' chaotic oscillation. In addition, the reasons behind the random phase's effect are explored. The nonautonomous system with phase perturbation can be considered as one of the simplest nonlinear systems displaying chaos phenomena. The perturbation disrupts the motion integrals and produces an unstable region in phase space. If the Chirikov condition is met, the unstable region takes the form of a chaotic area with rapid diffusion [19].

## VI. LOAD INDUCED CHAOS IN POWER SYSTEM

In (1988) [20], [21] considered a simple power system. The model is including a load bus connected to two generator buses. One of the generator buses has been defined as an infinite bus. While the second bus is characterized by the swing equation. A simple induction motor with a constant PQ load in parallel is used to mimic the load. Then load voltage and frequency are used as a simplified model of induction motor and provide the real and reactive power demands. The schematic model is shown in Fig. 4. They show the existence of chaos due to load variation.

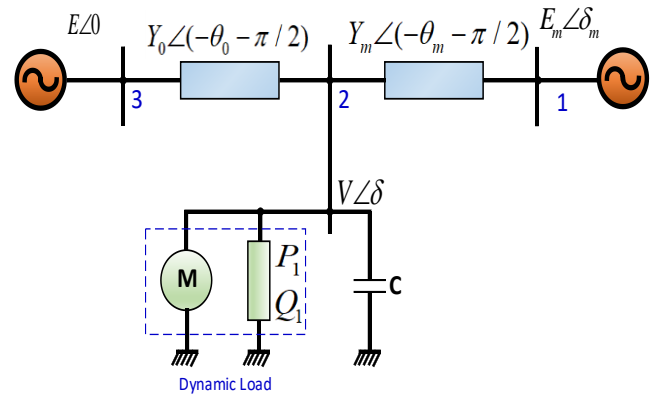


Fig. 4 A sample three-bus power system adapted from [22].

In (1993) [23] using computer simulations of a simple power system under a variety of loading circumstances, chaotic phenomena have been found. The authors studied the static (or local) bifurcation behavior of the power system under various loading conditions of reactive power. The following list summarizes the parameter values  $Q_1$  linked with these four different types of bifurcations:

- When  $Q_1 = 10.9461$ , subcritical Hopf bifurcation.
- When  $Q_1 = 10.8859$ , period-doubling bifurcation.
- When  $Q_1 = 11.3776$ , period-doubling bifurcation.
- When  $Q_1 = 11.4066$ , subcritical Hopf bifurcation.
- When  $Q_1 = 11.4106$ , saddle node bifurcation.

They focus their efforts and attention for  $Q_1$  values, which are in the vicinity of the two period-doubling bifurcations. And found chaos when  $Q_1 = 11.377$ .

## VII. MECHANICAL POWER INDUCED CHAOS IN POWER SYSTEM

K.G. Rajesh (1999) [24] investigated in an electrical power system model the existence of the bifurcations behavior. The dynamics of the generator are represented by a two-axis model where the field winding is considered to be on the  $d$ -axis while the damper winding is represented on the  $q$ -axis, as well as the excitation system. The load model is characterized by a dynamic load. The behavior of the power system is examined through:

- The generator input power,
- Load bus active and reactive power,
- AVR voltage reference.

These variables are considered as bifurcation parameters. Model refinement is found to result in considerable qualitative changes in system behavior. It is shown that quasiperiodic dynamics result from a torus-bifurcation. Moreover, as a consequence of cascades of period-doubling (PD) bifurcations, the system also can exhibit chaotic nature.

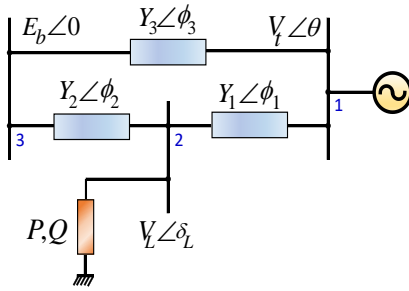


Fig. 5 Power system adapted from [25].

Z. Jing (2003) [26] analyzed an electrical power system including three buses. The input power to the generator ( $P_m$ ) considered as a bifurcation parameter, the system, due to includes nonlinear effects, has complicated dynamics that develop from static and dynamic bifurcations and lead to voltage collapse. The model analysis reveals different dynamical bifurcations types, including, cyclic-fold CB bifurcations, three Hopf bifurcations HB, period-doubling PD bifurcations, torus bifurcations TB. Moreover, it exhibits complex behaviors including periodic motion, period-doubling motions, quasi-periodic motions, phase-locked behavior. In addition to, two chaotic areas between two Hopf bifurcations, i.e. there are intermittency chaos and Hopf window.

### VIII. TIME-DELAY INDUCED CHAOS IN POWER SYSTEM

Time delay is unavoidable in realistic applications and exists commonly in the power systems measurement and control loops. Time delays in power systems were previously tolerated within an acceptable margin since feedback controllers were constructed using local information [27]. However, in current times, power systems have grown in size and complexity, with generators connected over long distances by systematic tie lines and exciter inputs coming from distant buses. Under such situations, the time-delay might range from tens to hundreds of milliseconds or more, resulting in unsatisfactory performance such as synchronism loss and power system instability.

M. Ling *et al.* in (2015) [28] explored the coexisting of attractors in a fourth-order time-delayed power system with various initial conditions for the first time. Using the analysis of bifurcation diagrams, Poincaré maps, power spectral density PSD, and phase portraits, for varying generator damping factors, mechanical power, the gain of the excitation system, and time delay. They revealed the characteristics of the time-delay in electrical power systems, including a discontinuous or jump bifurcation behavior. Furthermore, in the power system, the coexistence of two separate periodic trajectories and chaotic attractors with periodic orbits has been found. Therefore, the time delay can promote the system dynamics complexity, causing chaotic power oscillation and potentially voltage collapse. And voltage collapse may result in a blackout.

In (2005) [29] investigate the effect of time-delay in feedback control of a SMIB electrical power system. They concluded that time-delay can trigger chaos phenomenon in the power system.

### IX. SUBSYNCHRONOUS RESONANCE (SSR) INDUCED CHAOS IN POWER SYSTEM

Subsynchronous resonance or SSR is a phenomenon that occurs when a resonant situation exists between the transmission line and generation unit, resulting in an oscillation with frequencies lower than the power system's fundamental frequency (subsynchronous frequencies). This resonance phenomenon is termed subsynchronous resonance [30], [31]. During this case, series compensation boosts the transmission line's power transfer capabilities. At some compensation levels, however, Hopf bifurcation is exhibited. In the event of a traditional compensation (variable series capacitor) method, the system then goes into chaos via a torus breakdown scenario.

In (2003, 2004, 2013) [32]–[34], respectively, the scholars considered the first system of the IEEE second benchmark models of subsynchronous resonance, shown in Fig. 6. And the power system's stability when the compensation factor, defined as the ratio of the series capacitor's and inductor's reactances, is changed. In this article, the compensating factor is regarded as the bifurcation parameter. Results show that the system can exhibit chaotic oscillation due to SSR.

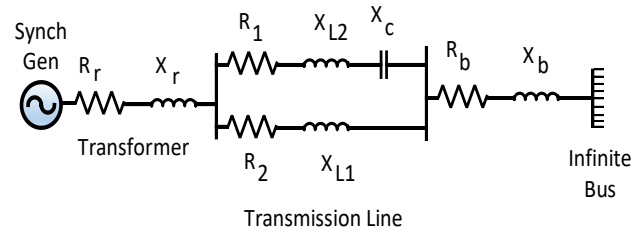


Fig. 6 Power system under study (System#1, IEEE second Benchmark Model of SSR).

In (2009, 2017) [35], [36] to explore the system's complicated dynamics, a bifurcation analysis is applied to the second system of the IEEE second benchmark model. It is addressed the scenario of ignoring the dynamics of both damper windings. The results reveal that the operating point loses stability via Hopf bifurcation as the compensating factor ( $X_c/X_L$ ) grows. Also, examined the effect of including damper windings to the generator's  $d$  – and  $q$  – axes.

### X. FERRORESONANCE INDUCED CHAOS IN POWER SYSTEM

Ferroresonance is caused due to traversing capacitance line of the system with a nonlinear area of transformer saturation curve due to several configurations like circuit breaker failure, line, and plant outage, voltage transformer connected to grading capacitor circuit breaker, and so on. The waveforms are distorted, and the frequency difference between two sites in the grid is increased. The frequency difference  $\Delta f$  causes a power oscillation with a swing frequency equal to  $\Delta f$ . The ferroresonance term was firstly coined in 1920 to characterize the phenomenon of two stable fundamental frequency operating points in a series capacitor circuit with a resistor, and nonlinear inductor.

In (1994) B.A. Mork [37] for the first time, ferroresonance was studied from the perspective of nonlinear analysis and chaos theory. They indicate that single-phase switching or interrupting can cause ferroresonant undervoltages or overvoltages in cable-fed transformer setups.

In (1995) by S. Mozaffari [38] a detailed study of numerous simulation findings, demonstrating that as losses drop and transformer magnetization nonlinearity rises the chance of chaos evolving. The chaotic solution of the system was investigated by altering the transformer core losses and the value of the source voltage.

In (2013) Radmanesh [39] employed chaos theory, phase plan analysis, bifurcation, and time-waveforms simulation are used for investigating ferroresonance. A light load or no-loaded power transformer is included in the proposed power system. A single-value two-term polynomial is used to represent the transformer core's magnetization curve. The dynamic behavior of a transformer in the event of ferroresonance, as well as non-linearity in the core loss, has been investigated. Some modes of ferroresonance oscillation have been derived too.

In (2019) Rezaei [40], discussed the severity of ferroresonance in power systems and classified it into four types such as fundamental, harmonic, quasi-periodic, and chaotic [41].

## XI. CONCLUSIONS

Different types of bifurcations and chaos phenomena, and a variety of electrical power systems configurations, have been reviewed in this paper. The work presented summaries of some articles by distinguished researchers in the field of nonlinear power system analysis. This review sheds light on the importance of studying the phenomenon of chaotic oscillation in power systems because it is one of the complex and extensive nonlinear systems. A set of causes of chaos were addressed, including limitation in the circuit of generators excitation, noise that is commonly present in electromechanical systems, changes in linear and non-linear loads, the effect of variations in the output power of the turbine unit providing the mechanical power to generators, and the effect of time delay in feedback systems. Furthermore, SSR and ferroresonance have been identified as two key contributors of chaotic oscillation in electrical power systems. This review gives vital information which is expected to advise new researchers and assist newcomers in seeing some of the most important results and gaining an appreciation of this broad topic.

## CONFLICT OF INTEREST

The authors have no conflict of relevant interest to this article.

## REFERENCES

- [1] B. A. Carreras, J. M. Reynolds-Barredo, I. Dobson, and D. E. Newman, "Validating the OPA cascading blackout model on a 19402 bus transmission network with both mesh and tree structures," 2019.
- [2] H. H. Alhelou, M. E. Hamedani-Golshan, T. C. Njenda, and P. Siano, "A survey on power system blackout and cascading events: Research motivations and challenges," *Energies*, vol. 12, no. 4, pp. 1–28, 2019.
- [3] L. L. Lai, H. T. Zhang, S. Mishra, D. Ramasubramanian, C. S. Lai, and F. Y. Xu, "Lessons learned from July 2012 Indian blackout," in *9th IET International Conference on Advances in Power System Control, Operation and Management (APSCOM 2012)*, 2012, pp. 1–6.
- [4] Y. Wang, M. Luo, Y. Xiao, and H. Chen, "Research on chaos phenomena in power system," *PEAM 2011 - Proc. 2011 IEEE Power Eng. Autom. Conf.*, vol. 2, no. 2, pp. 453–456, 2011.
- [5] N. Shrirao, A. Pandey, V. Chaurasia, R. Gupta, N. Modi, and R. Malpani, "A Review: Comparative Analysis of Chaotic Systems With Their Fractional Order Counterparts," *SSRN Electron. J.*, pp. 1–9, 2020.
- [6] W. Ditto and T. Munakata, "Principles and applications of chaotic systems," *Commun. ACM*, vol. 38, no. 11, pp. 96–102, 1995.
- [7] X. Zhou, S. Li, and Y. Ma, "The review of chaos research in power system," *Adv. Mater. Res.*, vol. 542–543, pp. 111–114, 2012.
- [8] N. Kopell and R. B. Washburn, "Chaotic Motions in the Two-Degree-of-Freedom Swing Equations," *IEEE Trans. Circuits Syst.*, vol. 29, no. 11, pp. 738–746, 1982.
- [9] F. M. A. Salam, "Chaos in the one generator system with excitation feedback," in *The 22nd IEEE Conference on Decision and Control*, 1983, pp. 360–364.
- [10] R.-L. Chen and P. P. Varaiya, "Degenerate Hopf bifurcations in power systems," *IEEE Trans. circuits Syst.*, vol. 35, no. 7, pp. 818–824, 1988.
- [11] H.-D. Chiang, "Application of Bifurcation Analysis to Power Systems," pp. 1–24, 2004.
- [12] J. Li and V. Venkatasubramanian, "Study of Hopf bifurcations in a simple power system model," *Proc. IEEE Conf. Decis. Control*, vol. 4, no. 4, pp. 3075–3079, 2000.
- [13] D. Q. Wei and X. S. Luo, "Noise-induced chaos in single-machine infinite-bus power systems," *Epl*, vol. 86, no. 5, 2009.
- [14] W. Ji and V. Venkatasubramanian, "Hard-limit induced chaos in a fundamental power system model," *Int. J. Electr. Power Energy Syst.*, vol. 18, no. 5, pp. 279–295, 1996.
- [15] A.-B. A. Al-Hussein, F. R. Tahir, and K. Rajagopal, "Chaotic Power System Stabilization Based on Novel Incommensurate Fractional-Order Linear Augmentation Controller," *Complexity*, vol. 2021, 2021.
- [16] V. Venkatasubramanian, "Coexistence of four different attractors in a fundamental power system model," *IEEE Trans. Circuits Syst. I Fundam. Theory Appl.*, vol. 46, no. 3, pp. 405–409, 1999.
- [17] Y. H. Qin and J. C. Li, "Random parameters induce chaos in power systems," *Nonlinear Dyn.*, vol. 77, no. 4, pp. 1609–1615, 2014.
- [18] Q. Ying-Hua, "Random-phase-induced chaos in power systems," *Chinese Phys. B*, vol. 19, no. 6, 2010.
- [19] B. V. Chirikov, "A universal instability of many-dimensional oscillator systems," *Phys. Rep.*, vol. 52, no. 5,

- pp. 263–379, 1979.
- [20] I. Dobson, H. D. Chiang, J. S. Thorp, and L. Fekih-Ahmed, “Model of voltage collapse in electric power systems,” *Proceedings of the IEEE Conference on Decision and Control*, pp. 2104–2109, 1988.
- [21] I. Dobson and H. D. Chiang, “Towards a theory of voltage collapse in electric power systems,” *Syst. Control Lett.*, vol. 13, no. 3, pp. 253–262, 1989.
- [22] A.-B. A. Al-Hussein, F. R. Tahir, and O. Boubaker, “Chaos Elimination in Power System Using Synergetic Control Theory,” in *2021 18th International Multi-Conference on Systems, Signals & Devices (SSD)*, 2021, pp. 340–345.
- [23] H. D. Chiang, C. W. Liu, P. P. Varaiya, F. F. Wu, and M. G. Lauby, “Chaos in a simple power system,” *IEEE Trans. Power Syst.*, vol. 8, no. 4, pp. 1407–1417, 1993.
- [24] K. G. Rajesh and K. R. Padiyar, “Bifurcation analysis of a three node power system with detailed models,” *Int. J. Electr. Power Energy Syst.*, vol. 21, no. 5, pp. 375–393, 1999.
- [25] A.-B. A. Al-Hussein, F. R. Tahir, A. Ouannas, T.-C. Sun, H. Jahanshahi, and A. A. Aly, “Chaos suppressing in a three-buses power system using an adaptive synergetic control method,” *Electronics*, vol. 10, no. 13, p. 1532, 2021.
- [26] Z. Jing, D. Xu, Y. Chang, and L. Chen, “Bifurcations, chaos, and system collapse in a three node power system,” *Int. J. Electr. Power Energy Syst.*, vol. 25, no. 6, pp. 443–461, 2003.
- [27] H. Jia, N. Guangyu, S. T. Lee, and P. Zhang, “Study on the impact of time delay to power system small signal stability,” *Proc. Mediterr. Electrotech. Conf. - MELECON*, vol. 2006, no. 1, pp. 1011–1014, 2006.
- [28] M. L. Ma and F. H. Min, “Bifurcation behavior and coexisting motions in a time-delayed power system,” *Chinese Phys. B*, vol. 24, no. 3, 2015.
- [29] H. K. Chen, T. N. Lin, and J. H. Chen, “Dynamic analysis, controlling chaos and chaotification of a SMIB power system,” *Chaos, Solitons and Fractals*, vol. 24, no. 5, pp. 1307–1315, 2005.
- [30] M. Bongiorno, *On control of grid-connected voltage source converters: mitigation of voltage dips and subsynchronous resonances*. Chalmers Tekniska Hogskola (Sweden), 2007.
- [31] A. A. Fouad and K. T. Khu, “Subsynchronous Resonance Zones in the IEEE" Bench Mark" Power System,” *IEEE Trans. Power Appar. Syst.*, no. 3, pp. 754–762, 1978.
- [32] A. M. Harb and M. S. Widyan, “Modern nonlinear theory as applied to SSR of the IEEE second benchmark model,” *2003 IEEE Bol. PowerTech - Conf. Proc.*, vol. 3, pp. 967–973, 2003.
- [33] A. M. Harb and M. S. Widyan, “Chaos and bifurcation control of SSR in the IEEE second benchmark model,” *Chaos, Solitons and Fractals*, vol. 21, no. 3, pp. 537–552, 2004.
- [34] M. S. Widyan, “Controlling chaos and bifurcations of SMIB power system experiencing SSR phenomenon using SSSC,” *Int. J. Electr. Power Energy Syst.*, vol. 49, no. 1, pp. 66–75, 2013.
- [35] A. M. Harb and M. M. Alomari, “Bifurcation and chaos theory as applied to SSR of the IEEE second benchmark model (system # 2) with the presence of the damper windings,” *Proc. 2005 Int. Conf. Sci. Comput. CSC'05*, vol. 6203, no. December, pp. 177–183, 2005.
- [36] M. M. Alomari, M. S. Widyan, M. Abdul-Niby, and A. Gheitasi, “Hopf Bifurcation Control of Subsynchronous Resonance Utilizing UPFC,” *Eng. Technol. Appl. Sci. Res.*, vol. 7, no. 3, pp. 1588–1594, 2017.
- [37] B. A. Mork and D. L. Stuehm, “Application of nonlinear dynamics and chaos to ferroresonance in distribution systems,” *IEEE Trans. Power Deliv.*, vol. 9, no. 2, pp. 1009–1017, 1994.
- [38] S. Mozaffari, S. Henschel, and A. C. Soudack, “Chaotic ferroresonance in power transformers,” *IEE Proc. Gener. Transm. Distrib.*, vol. 142, no. 3, pp. 247–250, 1995.
- [39] H. Radmanesh and G. B. Gharehpetian, “Ferroresonance suppression in power transformers using chaos theory,” *Int. J. Electr. Power Energy Syst.*, vol. 45, no. 1, pp. 1–9, 2013.
- [40] S. Rezaei, “Power Oscillation Due to Ferroresonance and Subsynchronous Resonance,” *Power Syst. Stab.*, 2019.
- [41] P. Ferracci, “Ferroresonance—Cahier technique Schneider no. 190,” *Groupe Schneider*, 1998.