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The Effect of Sample Size on the Interpolation Algorithm of Frequency Estimation

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Abstract
Fast and accurate frequency estimation is essential in various engineering applications, including control systems, communications, and resonance sensing systems. This study investigates the effect of sample size on the interpolation algorithm of frequency estimation. In order to enhance the accuracy of frequency estimation and performance, we describe a novel method that provides a number of approaches for calculating and defending the sample size for of the window function designs, whereas, the correct choice of the type and the size of the window function makes it possible to reduce the error. Computer simulation using Matlab / Simulink environment is performed to investigate the proposed procedure's performance and feasibility. This study performs the comparison of the interpolation algorithm of frequency estimation strategies that can be applied to improve the accuracy of the frequency estimation. Simulation results shown that the proposed strategy with the Parzen and Flat-top gave remarkable change in the maximum error of frequency estimation. They perform better than the conventional windows at a sample size equal to 64 samples, where the maximum error of frequency estimation is 2.13e-2 , and 2.15e-2 for Parzen and Flat-top windows, respectively. Moreover, the efficiency and performance of the Nuttall window also perform better than other windows, where the maximum error is 7.76×10-5 at a sample size equal to 8192. The analysis of simulation result showed that when using the proposed strategy to improve the accuracy of the frequency estimation, it is first essential to evaluate what is the maximum number of samples that can be obtained, how many spectral lines should be used in the calculations, and only after that choose a suitable window.

Keywords
Frequency Estimation, Interpolation Algorithm, Fast Fourier Transform.

I. Introduction

Currently, frequency estimation methods can be split into two groups: the frequency estimation algorithms based on the time domain, and the frequency estimation algorithms based on the frequency domain. The algorithms of the time domain include maximum likelihood algorithms (ML) [1–4] , algorithms of autocorrelation [5, 6], algorithms of linear prediction [7], and algorithms of the least squares [8]. However, these algorithms are challenging to utilize in real-time applications due to the significant amount of calculations needed. Recently, Discrete Fourier Transform (DFT) is employed to estimate the frequencies of signals, which requires even less computational effort, therefore, it is useful for real-time applications. However, it was shown that the DFT has critical constraints in estimating the precision frequencies for short-time signals [9–16]. The highest magnitude of the DFT samples is measured via tough search utilizing simple maximum search steps.The search of the precision allows proportional deviation of signal frequency from hard estimation by confirmed interpolation methods. In this study, we develop a novel and effective strategy aimed at evaluating the maximum number of samples and how many spectral lines should be obtained, after that, we can choose a suitable window to evaluate the frequency of signal, along with a comparison of the results obtained for generated signals to test efficiency. The rest of this paper is organized as follows. In the second section, the proposed algorithm is described. The results and discussion are explained in the third section, and the conclusion is shown in the last section.

II. Methodology of the Paper

The real sine wave paradigm is currently used in practical applications, and real sine wave estimation of the frequency is more complicated than complex sinusoidal signal because of the spectrum leakage from the negative signal parameter. Many researchers have provided their simple sine wave algorithms [17–25].The authors in reference [17] proposed an estimator similar to ML using spectrum matching. The technique avoids spectral derivation by comprising it in the signal's spectrum. However, this algorithm requires a comprehensive search that requires much computation. In study [18], it has been presented a new interpolation strategy relevant to the complex spectrum of several windows, a complex formula for finding the frequency of the component explained in equations (1) and (2) . This complex spectral strategy is less sensitive to spectral derivation than the modular methods. The frequency correction value is calculated as flowing:

$$\delta_1 = \pm\frac{\alpha_1}{\alpha_1 - 1}\delta_1 = \pm\frac{\alpha_1}{\alpha_1 - 1} \tag{1}$$

$$\delta_1 = \pm\frac{2\alpha_2 + 1}{\alpha_2 - 1} \tag{2}$$

where $\alpha_1 = \frac{X_{k-1}}{X_k}$ if $X_{k-1} \geqslant$, and

$$\alpha_2 = \frac{X_k}{X_{k+1}}, \quad \text{if} \quad X_{k-1} < X_{k+1} \tag{3}$$

There are three spectral lines' highest amplitudes: X_k , X_{k-1}, and X_{k+1}, as shown in Fig. 2, where δ is the offset of the fractional standardized frequency with range (-0.5; 0.5). The maximum spectral line, as well as two more spectral lines to its right and left, are used by Ding to conduct a fine search in [19]:

$$\alpha = \frac{X_{k+1} - X_{k-1}}{X_k + X_{k-1} + X_{k+1}} \tag{4}$$

In this study presents a precise frequency estimation of the real sine wave employing DFT. The suggested estimation has been tested with six windows (Parzen, Flat-top, Blackman, Kaiser, Bohman, and Chebyshev) to minimize undesirable effects resulting from spectral leakage caused by the FFT process. The estimation relies on interpolating the peak DFT spectral line and two DFT spectral lines and can be used with most windows. The general procedure of the interpolation estimator is explained in Fig.1. Let us consider the sinusoidal signal as the discrete sequence [6]:

$$p[n] = A\sin(2\pi F_0 n + \phi), \quad n = 0, 1, 2, \ldots, N-1 \tag{5}$$

Where A is the amplitude,F_0 is the frequency, ϕ is the initial phase of sine wave, and N is the sample size. If the number of periods of the sine wave are integer, consequently the frequency of signal can be calculated as below:

$$F_0 = \frac{F_s}{N}F_0 = \frac{F_s}{K} \tag{6}$$

Where K is the parameter of the discrete of the frequency index of the maximum in spectral lines DFT, and Fs is the sampling frequency as shown in Fig. 2. The frequency resolution [6]:

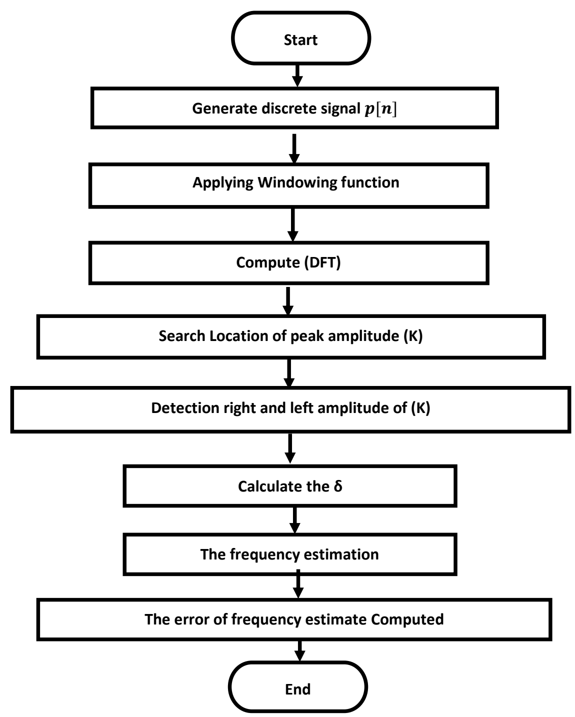

Fig. 1. Procedure of the general interpolation method.

$$\Delta F = \frac{F_s}{N} \tag{7}$$

ΔF is the distance between two spectral lines (frequency resolution). Applying the windowing function on $p[n]$:

$$P[n] = p[n] * w[n] \tag{8}$$

The windowing function Flattop, Parzen, and Bohman windows are used in this paper and explained in the next sections.

A. Flattop window

The Flattop window has a very low passband ripple. However this window drawback gives the low frequency resolution and the broad bandwidth. Flattop windows are summations of cosines as shown in equation (9):

$$w(n) = a_0 - a_1 \cos\left(\frac{2\pi n}{N-1}\right) + a_2 \cos\left(\frac{4\pi n}{N-1}\right) - a_3 \cos\left(\frac{6\pi n}{N-1}\right) + a_4 \cos\left(\frac{8\pi n}{N-1}\right) \tag{9}$$

The coefficients for this window are [20]:
$a_0 = 0.2155789$, $a_1 = 0.4166315$, $a_2 = 0.27726315$, $a_3 = 0.083578947$, $a_4 = 0.006947368$

B. Parzen window

Parzen Window is a piecewise-cubic approximations of Gaussian windows. This technique is usually applied to reduce side lobe of a sine wave levels, but it tends to have large scalloping loss. The Parzen Windows technique is explain mathematically in flowing equation [21]:

$$\begin{aligned} w(n) &= 1 - 6\left(\frac{|n|}{\frac{N}{2}}\right)^2 \pm 6\left(\frac{|n|}{\frac{N}{2}}\right)^3, \quad 0 \leqslant |n| \frac{N-1}{4} \\ w(n) &= 2\left(1 - \frac{|n|}{\frac{N}{2}}\right)^2 \quad \frac{N-1}{4} < |n| \leqslant \frac{N-1}{2} \\ & n = 0, 1, 2, \ldots, N-1 \end{aligned} \tag{10}$$

Where n is the number of data points.

C. Bohman window

Bohman window is the convolution of two half-duration cosine lobes. In the time domain, Bohman window is the product of the triangular window and the single cycle of a cosine with the term added to set the first derivative to zero at the boundary. The equation for computing the coefficients of Bohman window is explained in the flowing equation [21]:

$$w(x) = (1 - |x|)\cos(\pi|x|) + \frac{1}{\pi}\sin(\pi|x|) \quad -1 \leqslant x \leqslant 1 \tag{11}$$

Where x is a length vector of linearly spaced values generated using line space. The analysis program looked at the signal's spectral properties for many uses, including spectroscopy. DFT is as well often mention to as (FFT) and is the algorithm that perform DFT. In DFT, the sliding window technique is first utilized to the input sine wave as a discrete sequence $p[n]$. That is, the signal could be soft at it is ends. The selection of analysis window is an interesting developed field and affects the spectral resolution of the analysis. The DFT for the sine wave as a discrete sequence $p[n]$ is calculated by the flowing equation [21]:

$$X(k) = \frac{1}{N} \sum_{n=0}^{N-1} p(n) e^{-j2\pi kn/NT} \tag{12}$$

Where k is the DFT bin number in the range $0 \leqslant k \leqslant N–1$, and T is the sampling period .The crucial step in the analysis phase is to determine the amplitude and detected peaks. The proposed algorithm is similar to the idea of the different formulas available of Li, Dian. As shown in Fig. 1, the procedure of the general interpolation method of the computational steps start with producing the signal and applying the sampling process for the input signal with the number of samples in the range (16-512), thereafter taking the time window process to this sampled signal to minimize spectral dispersion, consequently determine the (FFT) of the output signal, after that calculate the sample that including the highest amplitude, which that comprise desired frequency. However, if the number of cycles captured in the sampling process is an integer number of cycles, subsequently that clarify the desired frequency is existing between the two samples that hold the highest amplitude, as demonstrate in Fig. 2. Then, using equation (15)

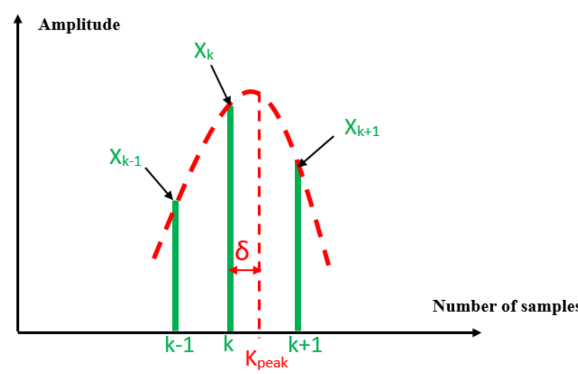

Fig. 2. The DFT spectrum applied for interpolation of the three points.

to calculate the variation between k and k + 1 or k -1 based on the amplitude with the highest value, we added it to k to determine the necessary frequency and assess the error using equations (16) and (17) [22]:

$$\Delta_1 = \frac{X_k}{X_{k-1}} \tag{13}$$

$$\Delta_2 = \frac{X_k}{X_{k+1}} \tag{14}$$

$$\delta = \frac{\frac{\Delta_2}{\Delta_1} - 1}{\frac{\Delta_2}{\Delta_1} + 1} \tag{15}$$

The signal frequency is calculated by [22]:

$$F_{Est} = \frac{\delta + K}{N} F_S \tag{16}$$

where $K' = \delta + K$.
The error of the frequency estimation is calculated by [22]:

$$Error = \frac{F_{Est} - F_c}{F_c} \tag{17}$$

III. Results and Discussion

To exemplify how the proposed algorithm work, we considered the parameters of simulations as follows: the phase in the range [-π , π] with step size 10^0 , Fs/Fc is equal to 4, the sampling frequency is chosen as (16,32,64,.....,512) Hz, and the sample size is equal to (16,32,64,.....,512) respectively. The most common interpolation methods (Parzen, Flat-top, Blackman, Kaiser, Bohman, Bartlett, Triangular, Nuttal, and Chebyshev) employed to estimate the frequency of the signal, along with the comparison of the results obtained for generated signal in order to test their performance.All the presented results are shown in Table I and Fig. 3. Table I compares the highest relative frequency estimation error and sample size across six windows. Kaiser window is used to implement the simulations' best outcomes when the sample number is small, such as N=16 and 32 samples. When using a Parzen, Flat-top window with medium sample size, such as N=64 samples, the efficiency of this algorithm is improved.The maximum systematic errors in these cases were 2.13e-2 and 2.15e-2, respectively. Chebyshev window was used to achieve the frequency estimation accuracy using the interpolation method algorithm, with 512 samples as the sample size and 2.44e-3 as the maximum systematic error. The findings also demonstrate that as the sample size increases, the suggested algorithm's maximum error gradually decreases. Fig. 4 shown graphs of the calculated maximum error values for the three spectral lines, three different windows (Bartlett, Triangular, and Nuttall), N= (16 to 512) samples, and a signal phase shift from -180 to +180.The Nuttall window performs better than other windows, where the maximum error =1.2e-3, while in Bartlett and Triangular, equal to 3.9e-3 and 2.9e-3, respectively, in Fig. 4. Fig. 4 shown graphs of the three different

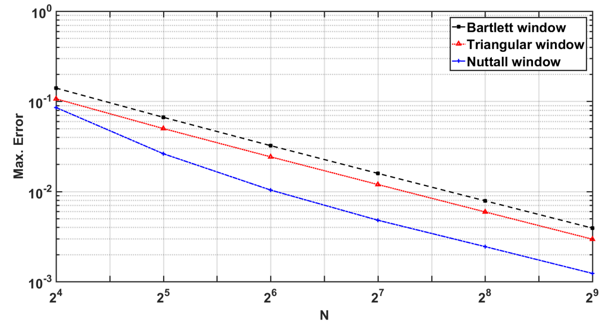

Fig. 3. Comparative analysis of the three windows.

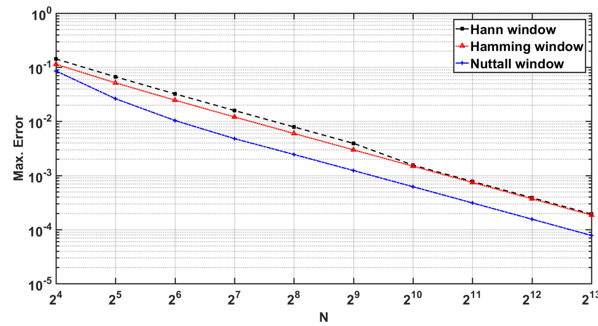

Fig. 4. Comparative analysis of the three windows.

windows (Hann, Hamming, and Nuttall) for N= (16 to 8192) samples. The Nuttall window also performs better than other windows, where the maximum error =7.76e-5, while Hann and Hamming are equal to 1.9e-4 and 1.8e-4, respectively, in Fig. 4. To demonstrate the effects of non-integer signal period numbers, consider the situation where N=16 and the Fc signal is between (3-4) Hz and 3.5 Hz, where the maximum frequency estimation error occurs. We have created a signal with a duration of N = 16, a step of 0.05, and a frequency spacing of 1 Hz between each component. Table II displays the Blackman window's impact on the frequency estimation mistake using Fc. The table shows that the highest frequency error occurs when the signal frequency approaches 3.5 Hz. A Blackman window was used, where the mistakes at N=16 were $1.4263x10^-1$.

IV. Conclusion

This study proposes a novel and effective algorithm based on the DFT samples of three point interpolation to estimate the fundamental frequency. The analysis of the simulation results shown that when using the proposed method, it is first necessary to evaluate what is the maximum number of samples that can be obtained, how many spectral lines should be used in the calculations, and only after that choose a suitable window.

TABLE I.
MAXIMUM ERROR FOR SIX WINDOWS

Maximum error for windows						
N	**Parzen**	**Flat-top**	**Blackman**	**Kaiser**	**Bohman**	**Chebyshev**
16	1.13e-1	1.53e-1	1.43e-1	8.76e-2	1.43e-1	1.16e-1
32	5.88e-2	7.28e-2	6.67e-2	5.57e-2	6.66e-2	5.88e-2
64	2.13e-2	2.15e-2	3.23e-2	2.99e-2	3.22e-2	3.03e-2
128	1.54e-2	1.75e-2	1.59e-2	1.53e-2	1.10e-2	1.53e-2
256	7.76e-3	8.72e-3	4.87e-3	7.74e-3	7.87e-3	7.75e-3
512	3.89e-3	4.35e-3	3.92e-3	3.80e-3	3.92e-3	2.44e-3

TABLE II.
THE SYSTEMATIC ERRORS FOR BLACKMAN WINDOW

FC	Error	FC	Error
3.05	8.8944e-5	3.55	1.5283e-1
3.10	-1.022e-2	3.60	1.1613e-1
3.15	-2.0592e-2	3.65	9.4090e-2
3.20	-3.1214e-2	3.70	7.5704e-2
3.25	-4.2403e-2	3.75	6.0203e-2
3.30	-5.4468e-2	3.80	4.6971e-2
3.35	-6.7762e-2	3.85	3.5511e-2
3.40	-8.2695e-2	3.90	2.5409e-2
3.45	-9.9762e-2	3.95	1.6319e-2
3.50	1.4263e-1	4.00	7.9387e-3

The results shown that the proposed algorithm performs better, especially when the Chebyshev window is used at a sample size = 512 samples. Furthermore, the proposed strategy offer high performance when the number of samples (N) is 128, and the Bohman window is used where the error drops to $(1.10x10^{-}2)$. The proposed algorithm could be apply as a framework for other interpolation algorithms, for improving overall performance in frequency evaluation.

V. CONFLICT OF INTEREST

The authors have no conflict of relevant interest to this article.

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