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Building A Control Unit of A Series-Parallel Hybrid Electric Vehicle by Using A Nonlinear Model Predictive Control (NMPC) Strategy

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Abstract
Hybrid electric vehicles have received considerable attention because of their ability to improve fuel consumption compared to conventional vehicles. In this paper, a series-parallel hybrid electric vehicle is used because they combine the advantages of the other two configurations. In this paper, the control unit for a series-parallel hybrid electric vehicle is implemented using a Nonlinear Model Predictive Control (NMPC) strategy. The NMPC strategy needs to create a vehicle energy management optimization problem, which consists of the cost function and its constraints. The cost function describes the required control objectives, which are to improve fuel consumption and obtain a good dynamic response to the required speed while maintaining a stable value of the state of charge (SOC) for batteries. While the cost function is subject to the physical constraints and the mathematical prediction model that evaluate vehicle's behavior based on the current vehicle measurements. The optimization problem is solved at each sampling step using the (SQP) algorithm to obtain the optimum operating points of the vehicle's energy converters, which are represented by the torque of the vehicle components.

KEYWORDS: Series-Parallel Hybrid Electric Vehicle (HEV), Nonlinear Model Predictive Control (NMPC), Planetary Gear Set (PGS), Energy Management Strategies (EMS), state of the charge (SOC).

I. Introduction

The series-parallel hybrid electric vehicle can operate either in series or parallel power flow mode due to the presence of a planetary gear set (PGS) that separates the power generated by the engine into mechanical and electrical paths [1]. The planetary gear set consists of three nodes called the ring gear, the sun gear, and the carrier gear where these nodes are connected directly to the engine (ICE), generator (MG1), and the vehicle's body respectively. While the motor (MG2) is also connected to a ring gear, which acts as a traction motor to propel the vehicle [2]. It also works as a generator to capture the kinetic energy resulting from the deceleration of the vehicle or when applying the brakes or when descending from a slope and this is called regenerative braking. Fig.1 shows the elements and connections of the planetary gear set and the configuration of the powertrain of the series-parallel hybrid electric vehicle.

As the energy is transmitted through the mechanical path from the engine to the planetary gear set (PGS) by the carrier gear and then it is transferred directly to the ring gear and reaches the vehicle after the energy passes through the reduction gear which connects the ring gear with the vehicle.

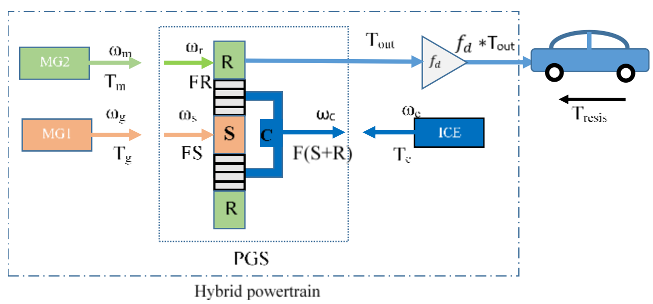

Fig.1: The powertrain and vehicle dynamic block

While the energy is transmitted through the electrical path from the engine to the Planetary Gear Set (PGS) by the carrier gear and then transmitted to the MG1 after passing through the sun gear, and then transferred as electrical energy to be stored in the on-board battery or used to feed the MG2 [3].

The series-parallel hybrid electric vehicle's control units that control the operation of the vehicle are based on one of the various energy management strategies (EMS)[4][5][6]. This strategy is responsible for managing the vehicle's energy represented by liquid fuel and batteries, in addition to the kinetic energy when decelerating or braking, as it works to improve fuel consumption and the vehicle's dynamic response by determining the operating points of the vehicle's energy converters (engine, motor, and generator)[7]. In this paper, the NMPC control strategy was used to manage the energy in a series-parallel hybrid electric vehicle. This strategy predicts the future behavior of the vehicle during a period called the prediction horizon. Therefore, it needs a mathematical model that describes the operation of the vehicle, and it also needs to create a cost function that expresses the control objectives of this strategy. To find the optimum value of the control inputs for a hybrid electric vehicle, this is done by solving its optimization problem, which is represented by the equation of cost subject to the mathematical prediction model and the physical constraints of the vehicle, which is solved by one of the methods of solving mathematical optimization problems. In this study, a nonlinear MPC controller provided by MATLAB was used to build this strategy, where this strategy aims to optimize the vehicle's fuel consumption and make the vehicle achieve the desired speed by the driver, as well as maintain the state of charge of the batteries at the desired value. Where this study can be used to learn how to build the mathematical model of the series-parallel hybrid electric vehicle as well as how to use the NMPC control strategy to manage this vehicle.

In [8] model predictive control (MPC) strategy was applied to a series-parallel HEV where was used linear mathematical prediction model to express the behavior of the series-parallel HEV which is used by this strategy to predict the future vehicle's behavior. The cost function used consists of three terms. The first term is the square of the difference between the actual and predicted torques of the wheels, and the predicted torque of the wheels is estimated using an adaptive recursive prediction algorithm that depends on the past and present torque of the drivetrain. The second term represents the square of the engine's fuel flow rate, and the last term represents the square of the batteries' equivalent fuel consumption. The cost function is solved by a Linear Quadrature Tracking (LQT) approach to obtain the control inputs values that achieve a minimum fuel consumption, minimizing the difference between the actual and predicted torque output of the drivetrain as well as maintaining the battery charge condition at the required level. In [9] is used model predictive control (MPC) strategy to manage the first level of the series-parallel hybrid electric vehicle's control unit. Since this unit consists of two levels, the first level finds the optimal values for both the speed and torque of the engine, which are references for the second control level. The standard linear MPC is used to solve the optimization problem in each sampling step by Quadratic Program (QP) approach to find the optimum values of both the speed and torque of the engine to get the minimum fuel consumption, reduce using the friction brake, and keep the state of the charge of the battery at the required level. In [10]the model predictive control (MPC) strategy was also applied to manage a control unit of the series-parallel HEV. Where the standard linear MPC was used to solve the optimization problem in each sampling step by MATLAB MPC toolbox to find the optimal values torque of the engine, motor, and generator in order to obtain the minimum fuel consumption, as well as maintain the state of the charge of the battery at the required level.

This paper has been organized in the following manner. Section II describes the architecture of the series-parallel HEV model used in this paper. Section III illustrates the dynamic equations for the powertrain and vehicle dynamic equations and the dynamic equations for the state of charge of the batteries in addition to describing how to express the rate of fuel flow equation in terms of both the speed and torque of the internal combustion engine. While section IV explains the MPC algorithm in details, and section V deals with the NMPC control strategy for a series-parallel HEV and how to formulate the optimization problem of a series-parallel HEV to achieve the objectives of the NMPC control strategy in addition that it also contains how to create the non-linear MPC controller block. Section VI explains Sequential Quadratic Programming (SQP) Algorithm. Section VI demonstrates the results of the simulation of the series-parallel HEV model, where its unit was based on the NMPC control strategy, while the last section discussed the results of the simulation and the future work of the study.

II. Architecture Of The Series-Parallel HEV Model

Building a model of a series-parallel hybrid electric vehicle to study its controller that was built based on a model predictive control. The approach used in this model is classified as a forward-looking approach, as the control unit depends on the required speed of the vehicle and the current speed of the vehicle in creating commands to produce torques through the drivetrain (the engine, motor, and generator), to obtain the required vehicle traction force[11]. This model consists of three parts, as shown in Fig.2.

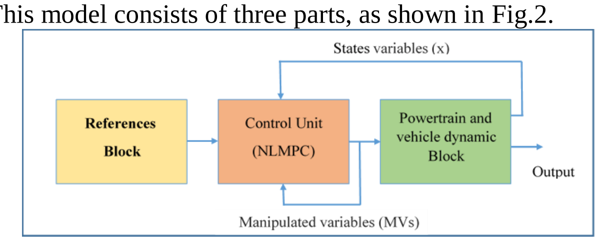

Fig.2: Architecture of the series-parallel HEV model

These parts are:-

- References block: - This block contains the driving cycle and the desired values or the target value for the state of charge (SOC) of the battery.
- Control unit: - The control unit was built depending on the NMPC control strategy. The driving cycle and the target state of charge of the battery are represented some of the control unit entries which come from the reference block. There is also feedback that only includes the manipulated variables (MVs) that were computed in the previous sampling step. There is also feedback that only includes the manipulated inputs that are computed for the current time step. In addition, the rest entries are the state variables, which come from the powertrain and vehicle dynamic block. While the engine torque, motor torque, and generator torque are the outputs of this block and go to the block of powertrain and vehicle dynamic.

- The block of powertrain and vehicle dynamic:-This block includes the dynamic equations that express the vehicle and the powertrain, as the powertrain includes the engine, motor, generator, batteries, and a planetary gear set. Engine torque, motor torque, and generator torque are the entries of this block that come from the control unit. While the state variables are transmitted from this block to the control unit, the outputs of this block are the vehicle speed and the battery charge state.

III. Dynamic Equations of a Series-Parallel HEV and Fuel Flow Rate equation

In general, the dynamic equations of the series-parallel hybrid electric vehicle can be divided into:-

A. The Powertrain and Vehicle Dynamic Equations

In the planetary gear set, the rotational speed of the ring gear ω_r , the sun gear ω_S and the carrier gear ω_C are governed by the constraints of kinematic equality at all times by the following relationship[12]:-

$$R\omega_r + S\omega_S = (R+S)\omega_C \quad (1)$$

The rest of the abbreviations used in the equations are found in Table I. The above equation can be written in terms of the engine speed ω_e (rad/sec), the motor speed ω_m (rad/sec), and the engine speed ω_g(rad/sec).

Where:

$$\left.\begin{array}{l}\omega_c = \omega_e \\ \omega_r = \omega_m \\ \omega_s = \omega_g\end{array}\right\}, \quad (2)$$

Then (1) becomes:

$$R\omega_m + S\omega_g = (R+S)\omega_e \quad (3)$$

The another constraint of the kinematic equality between the planetary gear set and the vehicle speed is that the rotational speed of the ring gear is a function of the vehicle speed and is represented by the following relationship:-

$$V_{veh} = r_w \frac{\omega_r}{f_d}, \qquad \therefore \omega_r = f_d \frac{V_{veh}}{r_w} \quad (4)$$

TABLE I
LIST OF ABBREVIATIONS AND THEIR VALUE

Abbr.	Definition	Units
A_f	Vehicle frontal area	2.16m^2
g	Gravitational constant	9.81m/s^2
ρ	Air density	1.18kg/m^2
C_d	Drag coefficient	0.26
m	Total mass of the vehicle	1200Kg
V_{veh}	Speed of the vehicle	m/s
θ	The slope of the road	0radians
C_t	Wheel rolling friction coefficient	0.01
T_{brk}	Friction brake torque	Nm
S	Number of teeth of the sun gear	30
R	Number of teeth of the ring gear	78
f_d	Final gear ratio	1.3
r_w	wheel radius	0.3m
Q_{batt}	Capacity of the battery	8.1A.h
P_{batt}	Battery power	watt
R_{batt}	Internal resistance of the battery	0.246Ohm
V_{oc}	Battery voltage	Volt
T_e	Engine torque	Nm
T_m	Motor torque	Nm
T_g	Generator torque	Nm

In the dynamic equations of powertrain and vehicle, the following assumptions are made:-

1. The carrier inertia, the ring inertia, and the sun inertia are neglected.
2. Only the longitudinal dynamics of the vehicle are considered.

Thus, the dynamic equations of powertrain and vehicle are[13]:

$$J_e\dot{\omega}_e = T_e - F(R+S) \quad (5)$$

$$J_g\dot{\omega}_g = FS + T_g \quad (6)$$

$$J_m\dot{\omega}_m = T_m + FR - T_{out} \quad (7)$$

$$m\,\dot{V}_{veh} = \{((T_{out}f_d + T_{brk})/r_w) - 0.5\rho C_d A_f V_{veh}^2 - mg(f_r\cos\theta + \sin\theta)\} \quad (8)$$

From (1)-(8) we obtain:

$$\dot{\omega}_m = \dot{\omega}_{m1} - \dot{\omega}_{m2} + \dot{\omega}_{m3} + \dot{\omega}_{m4} \quad (9)$$

$$\dot{\omega}_{m1} = ((A_1T_m)/(A_1B_1 - A_2B_2))$$

$$\dot{\omega}_{m2} = ((B_2T_e)/(A_1B_1 - A_2B_2))$$

$$\dot{\omega}_{m3} = ((T_g(A_1B_3 - A_3B_2))/(A_1B_1 - A_2B_2))$$

$$\dot{\omega}_{m4} = (A_1/(A_1B_1 - A_2B_2)\{((-0.5\rho C_d A_f r_w^3\ \omega_m^2)/f_d^3) - (mgf_r) + (T_{brk}/f_d)\}$$

$$\dot{\omega}_e = \dot{\omega}_{e1} + \dot{\omega}_{e2} + \dot{\omega}_{e3} - \dot{\omega}_{e4} - \dot{\omega}_{e5} \quad (10)$$

$$\dot{\omega}_{e1} = (T_e/A_1)$$

$$\dot{\omega}_{e2} = ((A_3T_g)/A_1) - ((A_2T_m)/(A_1B_1 - A_2B_2))$$

$$\dot{\omega}_{e3} = ((A_2B_2T_e)/(A_1(A_1B_1 - A_2B_2)))$$

$$\dot{\omega}_{e4} = (A_2T_g/A_1)((A_1B_3 - A_3B_2)/(A_1B_1 - A_2B_2))$$

$$\dot{\omega}_{e5} = (A_2/(A_1B_1 - A_2B_2)\{((-0.5\rho C_d A_f r_w^3\ \omega_m^2)/f_d^3) - (mgf_r) + (T_{brk}/f_d)\}$$

Where:-

$$A_1 = J_e + ((S+R)/S)^2 J_g$$

$$A_2 = -(R(R+S)/S^2)J_g$$

$$A_3 = (R+S)/S$$

$$B_1 = J_m + (R^2/S^2)J_g + (r_w/f_d)^2 m$$

$$B_2 = -R(S + R/S^2)J_g$$

$$B_3 = -R/S$$

B. Battery Dynamic Equations

One of the most important parameters in the energy management of hybrid electric vehicles is the battery state of charge (SOC), which represents a measure of the remaining electric energy in the battery, it is determined by[14]:

$$SOC = \frac{Q_{batt} - \int_{t_0}^{t} I_{batt}(\tau)d\tau}{Q_{batt}} \quad (11)$$

The battery current I_{batt} can be found after solve this equation[15]:

$$P_{batt} = V_{oc}I_{batt} - I_{batt}^2\ R_{batt} \quad (12)$$

Then

$$I_{batt} = -\frac{V_{oc} - \sqrt{{V_{oc}}^2 - 4R_{batt}P_{batt}}}{2R_{batt}}$$

The dynamic equation of the state of charge (SOC) can be obtained by taking the time derivative of (11)[16]:

$$\dot{SOC} = -\frac{V_{oc} - \sqrt{{V_{oc}}^2 - 4R_{batt}P_{batt}}}{2R_{batt}Q_{batt}} \quad (13)$$

Where the battery power can also be expressed[17]:

$$P_{batt} = \omega_g T_g + \omega_m T_m \quad (14)$$

After substitute (13) into (14), we get:-

$$S\dot{O}C = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_{batt}(\omega_g T_g + \omega_m T_m)}}{2R_{batt}Q_{batt}} \quad (15)$$

From (1) we obtain:-

$$\omega_g = A_3\omega_e + B_3\omega_m$$

Now substitute ω_g into (15)

$$S\dot{O}C = -\frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_{batt}((\frac{(R+S)}{S}\omega_c - \frac{R}{S}\omega_r)T_g + \omega_m T_m)}}{2R_{batt}Q_{batt}} \quad (16)$$

C. Fuel Flow Rate equation

Through the experimental data of the fuel flow rate obtained by (http://www.transportation.anl.gov/pdfs/HV/2.pdf), a mathematical relationship was formed between the fuel flow rate on the one hand, and on the other side, both speed and torque generated by the engine, by applying the multiple linear regression analysis method[18]. Where this method is used to form a mathematical model between a dependent variable represented here by the fuel flow rate, and several independent variables represented here by both the speed and torque generated by the engine, as shown in (17).

$$\dot{m}_f = a + b\,\omega_e + c\,T_e \quad (17)$$

Where the least square method is used to estimate coefficients of the regression, $a, b, and\ c$ in (17). Fig.3 represents the mathematical relationship to express the fuel flow rate in terms of both the rotational speed and the output torque of the engine, and it is noted in this figure that when the rotational speed and torque of the engine are increased, the fuel flow rate increases linearly.

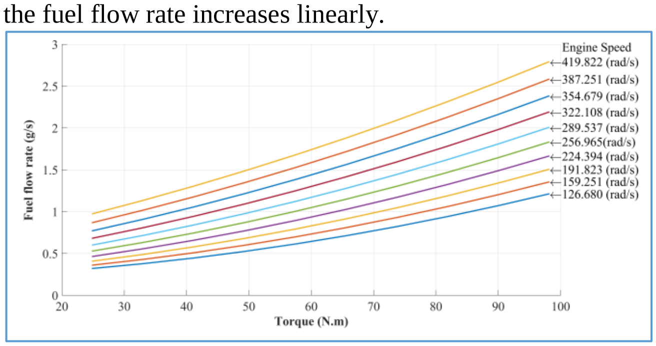

Fig.3: The fuel flow rate function

IV. MODEL PREDICTIVE CONTROL

The MPC algorithm is a process methodology (approach) used to control dynamic constrained systems[19], which is well suited to multivariate constrained operations. This algorithm is considered a class of computer control algorithms because it iteratively solves the optimization problem of this algorithm at each sampling step in order to find the optimal control input trajectory (manipulated variables (MVs)) of the plant. To achieve the control objectives on which this algorithm is built, it is formulated in the form of an optimization problem, which includes the cost function, which represents the objectives to be achieved by the algorithm, where the cost function is subject to predictions of the future behavior of the plant in addition to the plant's physical constraints. The predictions of the future behavior of the plant are obtained when using a process model, which is a mathematical model that describes the work of the plant, where the current measurements of the plant at the moment of sampling, represented by the values of state variables and optimal inputs (MVs), are used to predict the future behavior of the plant during a finite time interval called the prediction horizon. The prediction horizon can be defined as the future in which the algorithm can see the future behavior of the plant. At each sampling time, this algorithm works to find a solution to the optimization problem to obtain values of the optimal inputs trajectory, where only the first value of this trajectory is applied to the plant until the next sampling moment is reached. Because of the formulation of this algorithm and its dependence on process measurements at the moment of sampling to find the optimal inputs trajectory, it is considered as an open-loop controller [20].

Fig.4 shows the basic work of MPC, in which the MPC algorithm, at each sampling step, re-solves the optimization problem of open-loop control subject to system dynamics and constraints. Where the measurements obtained from the process model at current sampling time are used by the MPC algorithm to predict the future dynamics behavior of the plant y (•|k) over a prediction horizon Tp. Result of optimization problem solving is getting the optimal control input trajectory u (•|k), where only the first value of this trajectory is used to fed the next sampling step[21][22].

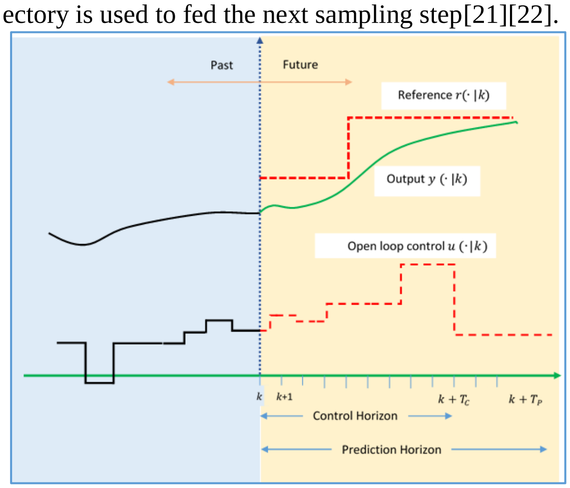

Fig.4: Basic principle of MPC

Due to the large number of computations resulting from predicting the behavior of system dynamics and solving the optimization problem at each sampling step over the prediction horizon, this definitely increases the demand for computation. The computational complexity can be greatly reduced by introducing a horizon called the control horizon Tc which is less than the prediction horizon. Where after the time interval of the control horizon Tc, the output of the controller is constant, where the value of the output of the controller is the value of the optimal control input at the sampling step of the control horizon Tc, assuming that the system has reached the steady-state[23], as shown in Fig.4.

If the predictions of the dynamic behavior of the plant are obtained from the equations of the nonlinear model, then the MPC in this case is called the Nonlinear Model Predictive Control (NMPC). Therefore, nonlinear predictive model control is an extension of linear predictive control

[24].Where the standard NMPC algorithm procedures are as follows [20]:-

- At each sampling step, use the current values of optimal control inputs (MVs) and measure or estimate the current values of the system state variables to use all these values to predict the future behavior of the plant across the prediction horizon. Calculate the open-loop optimal control from solving the optimization problem that is subject to dynamics of the system and constraints of the input and state over the prediction horizonTp.
- Calculate the optimal control inputs trajectory by solving the optimization problem, where the optimization problem is the cost function subject to predictions of the future behavior of the plant in addition to the physical constraints of the plant.
- Implement the first part of the optimal control inputs trajectory until the next sampling instant.
- Continue with step (1) when the next sampling step is reached.

In the MPC algorithm, the prediction trajectories for the state variables of the plant and the plant output are a linear function of both the current state variable and the optimal control input used in the current sampling step. Therefore, the solution of the optimization problem deals with the solvers that are efficient and high-performance. While the NMPC algorithm, the prediction trajectories for the state variables and outputs of the plant are a nonlinear function for the current state variable and the optimal control input used at the current sampling step. Thus the optimization problem becomes a nonlinear optimization problem and also known as nonlinear programming (NLP) problems, which needs a different approach (solvers) than in the MPC algorithm and where it is more computationally complex [25].

In NMPC the optimization problem is solved at every sampling step, which is represented by the cost function and the inequality and equality constraints as shown in below:-

$$\min_{U(k)} J = \mathcal{F}[Y(k), U(k)] \quad (18)$$

Subject to the following inequality constraints:

$$u_{min} > u(k + j\,|k) > u_{max},\ 0 > j > T_C - 1 \quad (19)$$

$$y_{min} > y(k + j\,|k) > y_{max}, 1 > j > T_P \quad (20)$$

In addition, the equality constraints:

$$x(k + j + 1|k) = F[x(k + j\,|k), u(k + j|k)],\ 0 > j > T_P - 1, \quad (21)$$

$$y(k + j\,|k) = h[x(k + j|k)] \quad , \quad 1 > j > T_P \quad (22)$$

Where:-

- $x(k + 1|k)$, $u(k + 1|k)$ are the state variable and optimal control input predicted at time $k + 1$ from measurements of the process model at time k respectively.
- $U(k)$ *and* $Y(k)$ are the optimal control inputs and outputs predicted from the process measurements of the model at time k respectively.

$$U(k) = [u(k|k),^T u(k + 1|k)^T, \dots, u(k + T_C - 1|k)^T] \quad (23)$$

$$Y(k) = [y(k|k),^T y(k + 1|k)^T, \dots, y(k + T_P\,|k)^T] \quad (24)$$

V. NMPC CONTROL STRATEGY FOR SERIES-PARALLEL HEV

In this study, the NMPC control strategy, which is one of the energy management strategies for hybrid electric vehicles, was chosen, as this strategy works to accomplish the tasks of the series-parallel HEV controller, and this strategy is built by formulating the optimization problem and solving it in one of the methods of mathematical optimization. The optimization problem includes the cost function subject to the nonlinear prediction model and physical constraints of the vehicle, where the cost function represents the objectives to be achieved by this strategy. In this study, the non-linear MPC control block was chosen to build and implement this strategy on the series-parallel HEV. The following is explained how to build the cost function and the non-linear MPC control

A. Formulation of the Optimization Problem for a Series-Parallel HEV

The optimization problem includes the cost function that represents the objectives to be achieved by the vehicle, and the cost function is subject to the predictions of the plant model that represents the equality constraints of the cost function. These predictions are obtained through the application of the mathematical model that describes the work of the vehicle based on the current values of the state variable and optimal control inputs (MVs) of the vehicle. The cost function is also subject to the physical constraints of the vehicle that represents the inequality constraints of the cost function. In this study, the cost function of the series-parallel HEV is formulated to minimize fuel consumption while ensuring that the vehicle can move at the speed required by the driver and maintain the state of the charge of the battery at the desired value. The optimization problem of the series-parallel HEV is shown in the following equations:-

$$\min_{U(k)} J = \int \|\Gamma(x, u)\|^2 dt \quad (25)$$

Subject to

1. Equality constraints:

$$\begin{cases} \dot{x} = f(x, u) \\ y = g(x, u) \end{cases} \quad (26)$$

Where:-

$$x = \begin{bmatrix} \omega_e \\ \omega_m \\ SOC \end{bmatrix}, \qquad u = \begin{bmatrix} T_e \\ T_m \\ T_g \end{bmatrix}, \qquad y = \begin{bmatrix} V_{veh} \\ SOC \end{bmatrix}$$

are the vectors of state, control inputs and tracking outputs respectively.

2. inequality constraints:

$$\begin{gathered} SOC^{min} \le SOC \le SOC^{max} \\ \omega_e^{min} \le \omega_e \le \omega_e^{max} \\ \omega_m^{min} \le \omega_m \le \omega_m^{max} \\ T_e^{min} \le T_e \le T_e^{max} \\ T_m^{min} \le T_m \le T_m^{max} \\ T_g^{min} \le T_g \le T_g^{max} \\ V_{veh}^{min} \le V_{veh} \le V_{veh}^{max} \end{gathered} \quad (27)$$

Where the superscripts "min" and "max" denote the lower and upper bounds of the parameters.

In (21) the integrand $\Gamma(x, u)$ is defined as:-

$$\Gamma(x,u)=\begin{bmatrix} w_{V_{veh}}(V_{veh}-V_{Ref}) \\ w_{SOC}(SOC-SOC_r) \\ w_{mf}(\dot{mf}) \end{bmatrix}$$

Where V_{Ref}, SOC_r are the required speed of the vehicle and the desired value of the SOC respectively, $w_{V_{veh}}$, w_{SOC} and w_{mf} are penalty weights. While the fuel flow rate equation ($\dot{mf}$) is described in (13).

The cost function of the series-parallel HEV is formulated in discrete-time form as:-

$$J=\sum_{i=0}^{T_P-1}((w_{V_{veh}}(V_{veh}(k+i+1)-V_{Ref})^2+(w_{SOC}(SOC\ (k+i+1)-SOC_r)^2+(w_{mf}(\dot{mf}(k+i+1)))^2) \quad (28)$$

Where: - i is represented sampling time.

B. Building NMPC control strategy for series-parallel HEV by using the non-linear MPC control block

A series-parallel HEV model was created using MATLAB whose controller was built based on the NMPC control strategy using the non-linear MPC control block provided in the MPC Toolbox in MATLAB SIMULINK®. This block is based on calculating the optimal control trajectory over the prediction horizon (Tp) by solving the nonlinear optimization problem which includes the nonlinear objective function subject to the nonlinear predictions of the future behavior of the plant and physical constraints of the plant. To implement this block, the number of the state variables, inputs control, and outputs concerning the predictive model of the plant is defined. In this study, the predictive model of the plant contains [26]:-

- Three state variables, which are the engine speed ω_e (rad/sec), motor speed ω_m (rad/sec), and SOC.
- Three inputs, which are the engine torque T_e (N.m), motor torque T_m (N.m), and generator torque T_g (N.m)
- Two outputs, which are Vehicle velocity (km/h), and SOC.

After defining the number of variables related to the predictive model of the series-parallel HEV the following will be specified:-

1. The dynamic states functions (time derivative of state functions) for a nonlinear prediction model, where the dynamic state equations in this study are motor speed $\dot{\omega}_m$ (rad/sec), engine speed $\dot{\omega}_e$ (rad/sec), and the state of charge $S\dot{O}C$ of the battery as shown in (9), (10), and (16) respectively. The computational efficiency of the controller becomes better when using an analytical Jacobian for dynamic states functions, and when not using an analytical Jacobian, the controller calculates the Jacobian by numerical perturbation. The Jacobian of the state functions are[26]:-

$$\nabla_x f=\frac{\partial f}{\partial x}=\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{n_x}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{n_x}}{\partial x_1} & \cdots & \frac{\partial f_{n_x}}{\partial x_{n_x}} \end{bmatrix}, \text{ and}$$

$$\nabla_u f=\frac{\partial f}{\partial u}=\begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_{n_u}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_{n_x}}{\partial u_1} & \cdots & \frac{\partial f_{n_x}}{\partial u_{n_u}} \end{bmatrix} \quad (29)$$

Where:-

f_1 to f_{n_x} are dynamic states functions of the model,
x_1 to x_{n_x} are states variables of the model,
u_1 to u_{n_u} are manipulated variables of the model.

2. The outputs functions for a nonlinear prediction model are vehicle velocity (km/h) is defined by (4), and SOC is represented by the third state variable. An analytical Jacobian was also used to improve the efficiency of the computational.
3. The non-linear MPC controller block needs to define a cost function that represents the objectives for which the controller is built or to be achieved by the controller. The cost function of any study when using a nonlinear MPC controller block can be represented by either using the standard cost function that consists of four terms where each term describes an aspect of the controller performance as shown in the following equation:-

$$J(z_k)=J_y(z_k)+J_u(z_k)+J_{\Delta u}(z_k)+J_\varepsilon(z_k) \quad (30)$$

Where the standard cost function terms are output reference tracking $J_y(z_k)$, manipulated variable tracking $J_u(z_k)$, manipulated variable move suppression $J_{\Delta u}(z_k)$, and constraint violation $J_\varepsilon(z_k)$ respectively. If the standard cost function cannot represent the cost function of a particular study, a function called the custom cost function can be built so that it contains terms that are not present in the standard cost function. Sometimes it is required to combine the standard cost function and a custom cost function to represent the control cost function. In this study, both the standard cost function and the assigned cost function are used to express the cost function of the series-parallel HEV shown in (28) is expressed by dividing it into two parts. The first part of the cost function illustrates making the vehicle move at the desired speed while maintaining the state of the charge of the battery at the desired value, which is represented by the first term of the standard cost function, which is called output reference tracking. The rest of the terms of the standard cost function are eliminated by making the penalty weight zero for each of them. While the second part of the cost function explains minimizing fuel consumption as it is represented by the custom cost function, which is the fuel flow rate equation (17).
4. Define the physical constraints of the system, which consist of the standard bounds on states, inputs, and outputs as shown in (27). While the equality and inequality of custom constraints were not used in this study.

After providing the previously mentioned requirements for the design or construction of a nonlinear MPC control block for the series-parallel HEV, this block calculates at each sampling step the optimal control inputs (engine torque T_e, motor torque T_m, generator torque T_g). The optimal control input is the solution of the nonlinear optimization problem represented by the cost function (28) subject to (26) and (26) but after formulating both (26) and (27) in the form of a discrete time as shown in (21) and (22) respectively. Where the nonlinear MPC controllers solve the nonlinear optimization problem using Sequential Quadratic Programming Algorithm (SQP), the following is an explanation of the SQP algorithm.

VI. Sequential Quadratic Programming Algorithm (SQP)

The SQP algorithm is considered one of the most important and successful methods for solving numerical constrained nonlinear optimization problems[27].This algorithm is based on forming the sub-problem of Quadratic Programming (QP) at each main iteration and using the resulting solution from this sub-problem to form the QP sub-problem at the subsequent iteration[28], where Fig.5 shows the flowchart of this algorithm. In general, this algorithm transforms a constrained nonlinear optimization problem into a series of successive iterations of quadratic programming (QP) sub-problems[29]. The basis of this algorithm is solving the nonlinear equations of the Karush-Kuhn-Tucker (KKT) optimality conditions equation of the constrained nonlinear optimization problems using Newton's numerical methods to solve these equations. Where it was found that this basis corresponds to solving the result of generating the sub-problem of Quadratic Programming (QP) iteratively, that is, at each iteration[27].

To implement the SQP algorithm for the following constrained nonlinear optimization problem:-

$$\textbf{Find } \boldsymbol{x} \textbf{ which minimizes } \boldsymbol{F}(\boldsymbol{x}) \quad (31)$$

Subject to

$$G_i(x) = 0 \qquad (i = 1,\ldots,m_e)$$

$$G_i(x) \leq 0 \qquad (i = m_e + 1,\ldots,m)$$

Where m_e and m are the number of equality constraints and number of constraints of the problem respectively. The Quadratic Programming (QP) sub-problem of this problem at k iteration is formulated as:-

$$min_s \ \frac{1}{2}\ S^T B_k S + \nabla F(x_k)^T S \quad (32)$$

$$\nabla F(x_k)^T S + G_i(x_k) = 0 \qquad (i = 1,\ldots,m_e)$$

$$\nabla F(x_k)^T S + G_i(x_k) \leq 0 \qquad (i = m_e + 1, \ldots, m)$$

Where S is search direction. As it is clear that the QP sub-problem () needs to find B_k, which is positive definite the approximate Hessian matrix of the Lagrangian function,

$$L(x,\lambda) = \boldsymbol{F}(\boldsymbol{x}) + \sum_{i=1}^{m} \lambda_i \cdot G_i(x) \quad (33)$$

Where λ_i is the Lagrange multiplier. The QP sub-problem also needs to make nonlinear constraints linear using the Taylor series approximation.

In the non-linear MPC control block, the QP sub-problem is solved using the active set strategy which needs an initial guess feasible for the QP sub-problem to find the direction of the search at the current iteration S_k and in addition to finding Lagrange multipliers λ_i, this solution contributes to the formation of the next iteration as shown below[30][31]:-

$$x_{k+1} = x_k + \alpha_k S_k \quad (34)$$

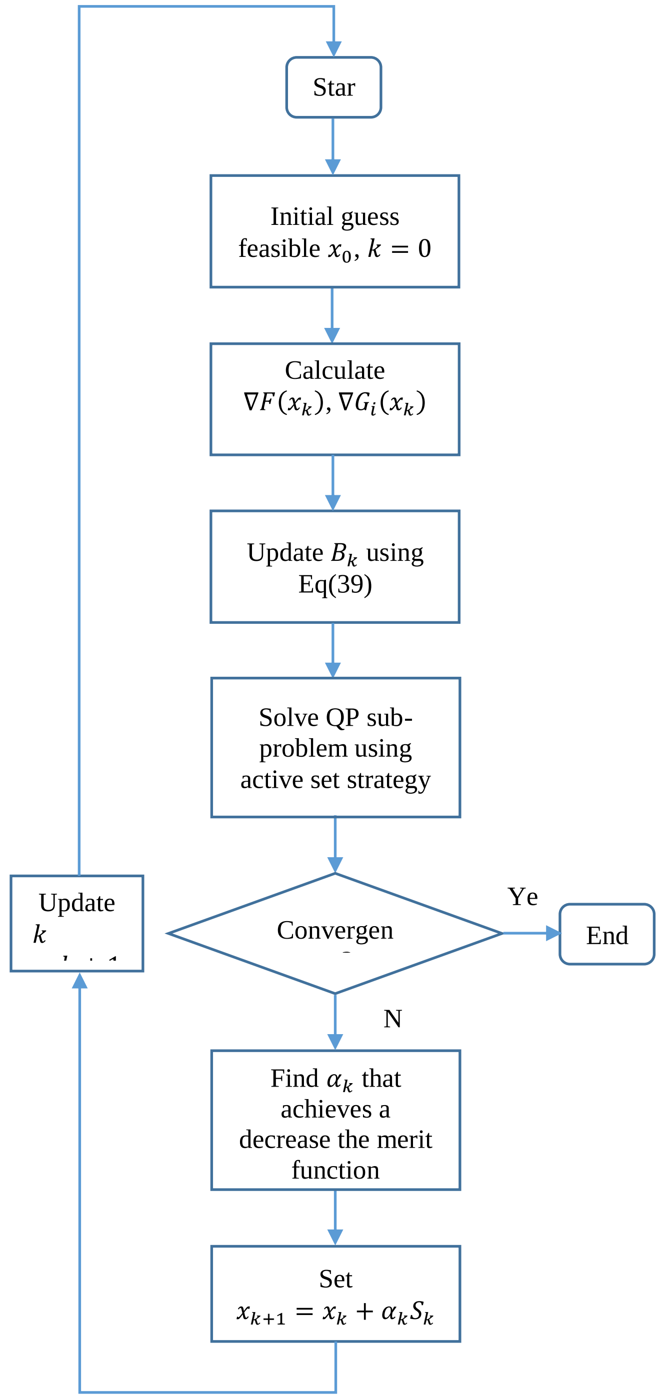

Fig.5: Flowchart of the SQP algorithm.

Where the step length parameter α_k is chosen in a way that achieves a decrease in the following merit function[27]:-

$$\Psi(x) = \boldsymbol{F}(\boldsymbol{x}) + \sum_{i=1}^{m_e}(r_i \cdot G_i(x)) \sum_{i=m_e+1}^{m}(r_i \cdot max[0, G_i(x)]) \quad (35)$$

Where r_i is the penalty parameter:-

$$r_i = (r_{k+1})_i = max_i\left\{\lambda_i, \frac{(r_k)_i+\lambda_i}{2}\right\} \ (i = 1,\ldots,m) \quad (36)$$

The initial values of the penalty parameters r_i:-

$$r_i = \frac{\|\nabla F(x)\|}{\|\nabla G_i(x)\|} \quad (37)$$

As for B, it is updated at major iterations using the Broyden Fletcher Goldfarb Shanno (BFGS) method as shown in the following equation[27][30]:-

$$B_{k+1} = B_k + \frac{q_k q_k^T}{q_k^T b_k} - \frac{B_k\, b_k b_k^T B_k^T}{b_k^T B_k b_k} \quad (38)$$

Where

$$b_k = x_{k+1} - x_k \quad (39)$$

$$q_k = \left(\nabla F(x_{k+1}) + \textstyle\sum_{i=1}^{m} \lambda_i \cdot \nabla G_i(x_{k+1})\right) - \left(\nabla F(x_k) + \textstyle\sum_{i=1}^{m} \lambda_i \cdot \nabla G_i(x_k)\right) \quad (40)$$

Since the solution of a nonlinear optimization problem has many solutions, it is difficult to find a solution to this problem unless we start with guess points that fall within the feasible solution regions that enable the SQP algorithm to make the first iteration to solve the optimization problem. In subsequent iterations, the predicted state variables and the optimal control inputs (control interval) from the previous step act as initial guesses for solving the optimization problem at that iteration. This is why feedback is built for this block, as shown in Fig.2. So when starting to solve a series-parallel HEV optimization problem, the guess torque values of the vehicle's energy converters are chosen and these values should be determined within the possible solution regions.

VII. Simulation Results

The model of the series-parallel HEV with the specifications mentioned in Table I was built and simulated using the MATLAB / Simulink (2019b) environment. As mentioned previously, this vehicle model consists of three blocks. The first block values represent the reference values or the values to be achieved by the outputs of this vehicle model, where the New European Driving Cycle (NEDC) was chosen to represent the required speed of the vehicle during the vehicle's trip. As for the second reference value, it is the SOC value, which represents the value required for the state of charge of the batteries during this trip, as this value was chosen to be 65% of the full value of the charging state of the batteries. Where this SOC value allows the battery to store the vehicle's kinetic energy captured from the vehicle's deceleration, as well as the possibility to equip the motor with electrical energy[32].

The second block, which represents the control unit of the series-parallel HEV model, was created by MATLAB Model Predictive Control toolbox using a nonlinear MPC controller block. This block needs to specify the number of the state variables, the manipulated variables, and the outputs where in this study are 3, 2, and 3 respectively, it also needs to specify the controller sample time, prediction horizon, and control horizon are determined by the following values 0.1 sec, 10, and 5 respectively. When creating the Nonlinear MPC controller block, a mathematical model to represent a series-parallel HEV is required to predict the vehicle's future behavior, where this model includes the engine speed ω_e (rad/sec), motor speed ω_m (rad/sec), and SOC are designated as the state variables. While the engine torque T_e (N.m), motor torque T_m(N.m), and generator torque T_g(N.m) are set as manipulated variables (MVs), and each the vehicle velocity (km/h) and SOC are the plant outputs.

To implement the Nonlinear MPC controller block needs to define the state equations of the nonlinear plant model ($\dot{\omega}_m$, $\dot{\omega}_e$, and $\dot{SOC}$) which are (9), (10), and (16) respectively, and also need to determine their Jacobian equations using (29). Also, this block needs the output equations of the nonlinear plant model (the vehicle velocity and SOC) and their Jacobian equations are determined by the following (29). In addition to setting the constraints for each of the state variables, the manipulated variables (MVs), and the output variables, as in (27). The second block also needs the values of the manipulated variables from the previous sampling step and the values of the state variables received by the third block for the purpose of finding the optimal torques values for each of the engine, motor, and generator to apply them to the third block, which represents the vehicle model (the powertrain and vehicle dynamic equations).

The aim of this simulation is to study the possibility of making the vehicle drive the required speed and also make the state of charge of the battery at the desired value, and that this is all done with minimal fuel consumption To accomplish this, we construct the objective function as shown in (28), since the objective function is composed of two parts where the first part of the objective function is represented by the first part of the standard cost function(30), which is called output reference tracking. In order to implement this part, the reference values for each of the outputs variables (vehicle speed and SOC) are received from the first block and the weight coefficients of the output variables are determined where the values 100 and 200 are chosen for each of the weight coefficients for the output variables (vehicle speed and SOC) respectively. As for the second part of the objective function, it is the custom cost function shown in (17).

After completing the simulation of the series-parallel HEV model, the results obtained were good because the vehicle control unit was able to achieve the desired objectives. Fig.6 shows the state variables are the engine speed ω_e (rad/sec), motor speed ω_m (rad/sec), and SOC. where it is observed that the state variable SOC changes between two values (65.2%) and (64%) during the driving cycle and this is a good result in tracking the desired value (65%), and this change is considered a small, insignificant change. While the outputs of the control unit are the optimal values of the manipulated variables (MVs) (engine torque (Nm), motor torque (Nm), and generator torque (Nm)) which are obtained by solving the optimization problem (minimizing the cost function subject to the nonlinear prediction model and physical constraints of the vehicle). Where Fig.7 represents these

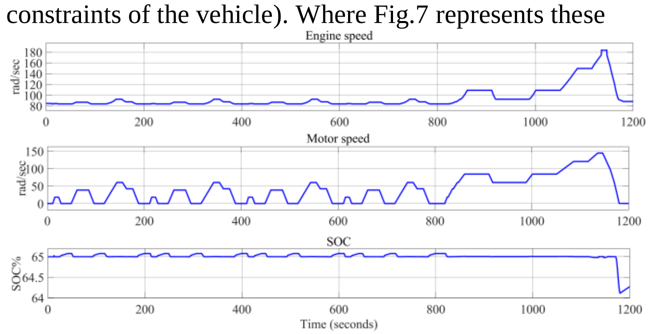

Fig.6: The state variables trajectory

these optimal values that are applied to the engine, motor, and generator to drive the vehicle in order to achieve tracking of each of the reference values of the vehicle speed and SOC as well as focus and attention to obtain the best possible minimization in fuel consumption.

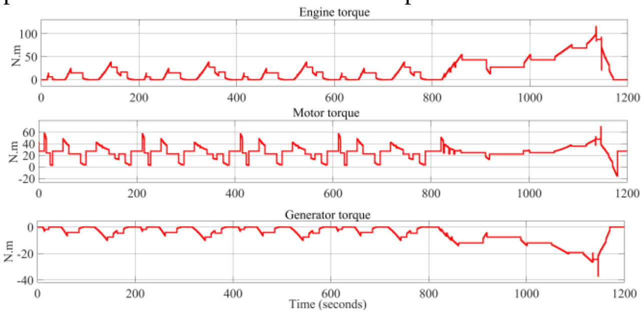

Fig.7: The trajectories of the optimal manipulated variables

It is noted that the tracking and dynamic response of the reference speed of the series-parallel HEV is good as shown in Fig.8, where the series-parallel HEV model was able to travel 10.9 km and the total fuel used was 0.4482 liters during the New European Driving Cycle (NEDC), and it is considered a good result in terms of improving fuel consumption. In the end, it can be said that a model of the hybrid electric vehicle was able to achieve the objectives required of it.

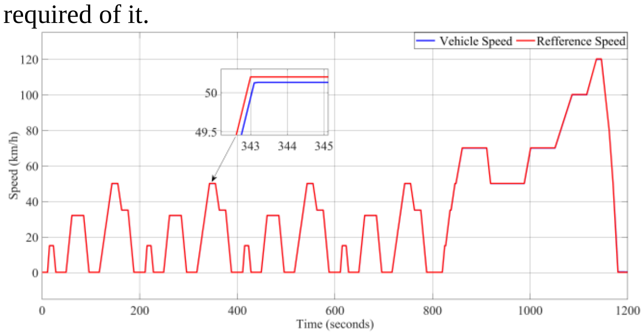

Fig.8: Series-parallel HEV speed by using NMPC strategy

VIII. Conclusions

To implement the NMPC control strategy for power management among the hybrid electric vehicle's energy converters, the nonlinear optimization problem must first be appropriately created, which includes the cost function subject to the physical constraints of the vehicle and the mathematical prediction model that is used to evaluate the vehicle's future behavior based on the vehicle's current measurements which are state variables and control inputs. In this study, this strategy improved the fuel consumption and dynamic performance of the vehicle as well as maintained the state of charge (SOC) of the battery at the desired value, they are formulated in the cost function. Secondly, this strategy solves the constrained nonlinear optimization problem by using the SQP algorithm at each sampling step to find the optimum values torque of the engine, motor, and generator to provide the power required to drive the vehicle. This strategy was able to achieve the required objectives of making the vehicle go at the required speed with the best fuel consumption, in addition to making the value of the state of the charge of the battery at the desired value. This indicates that the strategy was able to supply the motor with electrical energy, either the generator or the kinetic energy captured from the deceleration of the vehicle so that the state of the charge remains close to the desired value. Although this strategy makes many calculations, it was able to implement all the optimization tasks required of it well due to its ability to find the optimal operating points for energy converters in this vehicle. It is possible to benefit from this study by comparing the NMPC control strategy with other energy management strategies for the same vehicle, in addition to the possibility of using the mathematical model equations for this vehicle in other studies.

Conflict of Interest

The authors have no conflict of relevant interest to this article.

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