

## *Chaotic Characteristics of Vertical Cavity Surface Emitting Lasers Subject to Optoelectronic Feedback*

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### **Abstract:**

In this study, the chaotic dynamics observed from a vertical cavity surface emitting lasers ( VCSELs ) subject to delayed optoelectronic feedback are investigated. The theoretical investigation is performed by using a MATLAB software package. The nonlinear dynamics of a VCSEL are examined using a single – mode rate equations model. The key role played by system parameters such as delay time and the feedback strength on laser chaotic dynamics is addressed.

### المميزات التخبطية لليزرات الانبعاث السطحي ذات الفجوة العمودية لتبعية التغذية المرتدة الالكترونية الضوئية

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الخلاصة:

في هذه الدراسة تم عرض الديناميكية التخبطية ( Chaotic Dynamics ) لليزرات الانبعاث السطحي ذات الفجوة العمودية (VCSEL) بتبعية التغذية المرتدة الالكترونية الضوئية ذات التأخر الزمني ( Delayed Optoelectronic Feedback ). التحليل النظري تم تنفيذه باستخدام برنامج MATLAB. الديناميكية اللاخطية فحصت من خلال حل نموذج معادلات التغيير الزمني الخاصة بالليزر ( Laser Rate Equations ). لقد تم تقييم الدور الذي تلعبه بارامترات المنظومة مثل ( التأخر الزمني، شدة التبعية المرتدة ) في تعقيد الديناميكية التخبطية لليزر.

### I-Introduction:

Studies of chaos in nonlinear electrical circuits and semiconductor lasers have shown that chaotic signals generated in these systems can be used as a "carrier" for information transmission [1 - 3]. Time delay systems are widely used as generators of chaos in applications such as chaos communication [ 4 ]. Also, delayed feedback semiconductor laser have the potential to generate a chaotic dynamics.

A VCSELs are promising optical sources which can actively be used in the optical communication systems. The short cavity and hence the single spatial longitudinal mode operation represents one of the important advantages of this laser [5,6]. The VCSELs subject to delayed optoelectronic feedback produce nonlinear dynamical behavior. This is one way to generate high - frequency chaos and has in comparison to optical feedback, the advantage of being phase insensitive. The analysis of the dynamical behavior enables us to understand and improve the performance of semiconductor lasers in applications where feedback occurs, such as telecommunications.

A closed - loop optoelectronic system is considered, as illustrated in Fig.1. The VCSEL is driven by a dc bias current  $I_b$ , and the light power from a laser enters a photodetector which produces a current  $I_F$ , proportional to the light power, that then summed back into the power source that drives the laser. This process causes a delayed feedback due to the time taken by the electrical and optical signals to travel around the feedback. The delay time may be significant enough to create the nonlinear dynamics.

### II-Mathematic model of the optoelectronic device:

#### A-Source: The VCSEL

The dynamical equations that model the laser system are as follows [ 7 ]:

$$\frac{dP}{dt} = [G(N) - \gamma]P + R_{sp}(N) \quad \dots(1)$$

$$\frac{dN}{dt} = \frac{I_0 + I_F(t - \tau)}{q} - \gamma_c(N)N - G(N)P \quad \dots(2)$$

where P and N , respectively, the number of photons and number of carriers. Other parameters appeared in equations. ( 1 ) and ( 2 ) are defined as follows:

The rate of stimulated emission rate in the active region can be expressed as:

$$G(N) = \Gamma v_g a_0 \left( \frac{N}{V} - n_0 \right) \quad \dots(3)$$

where  $\Gamma$  is the optical confinement factor and can be written as :

$$\Gamma = \frac{d}{L} \quad \dots(4)$$

with d and L as the thickness of active region and the length of the cavity, respectively,  $v_g$  is the group velocity of the light,  $a_0$  is constant linear gain coefficient,  $n_0$  is the carrier density at transparency, and V is the volume of the active region and can be written as :

$$V = \frac{\pi}{4} D^2 d \quad \dots(5)$$

where  $D$  is the diameter of active region.

The photon decay rate can be expressed as:

$$\gamma = v_g [\Gamma \alpha_{ac} + (1 - \Gamma) \alpha_{ex} + \alpha_m] \quad \dots(6)$$

where  $\alpha_{ac}$  and  $\alpha_{ex}$  are the scattering loss in the active and external region respectively. The mirror loss  $\alpha_m$  is calculated by:

$$\alpha_m = \frac{1}{2L} \ln \frac{1}{\sqrt{R_f R_r}} \quad \dots(7)$$

where  $R_f$  and  $R_r$  are the power reflectivity of front and rear facets, respectively.

The term  $R_{sp}(N)$  is the spontaneous emission rate and can be written as:

$$R_{sp}(N) = \frac{\Gamma \lambda^4 B N^2}{4 \pi^2 \mu \mu_g \Delta \lambda_{sp} V^2} \quad \dots(8)$$

where  $\lambda$  is the light wavelength,  $\Delta \lambda_{sp}$  is the spectral width of the spontaneous emission,  $B$  is radiative recombination coefficient,  $\mu_g$  and  $\mu$  are the group refractive index and refractive index of the active region.

The carrier recombination rate  $\gamma_c(N)$  is given by:

$$\gamma_c = A_n + B \left( \frac{N}{V} \right) + C \left( \frac{N}{V} \right)^2 \quad \dots(9)$$

where  $A_n$  and  $C$  are surface recombination rate and nonradiative recombination coefficient, respectively.

Also  $I_0$ , and  $I_F(t - \tau)$ ,  $\tau$ , and  $q$  are, respectively, the dc bias current, the delayed feedback current, the parameter of the delay time, and the electron charge.

### B-Electronic Feedback loop

The electronic feedback loop connecting the photodetector and diode laser introduces a time delay. The linear feedback loop is described by [ 8 ]:

$$I_F = \xi \frac{\eta q}{h\nu} P_0 \quad \dots(10)$$

where  $\xi$  is the parameter of feedback strength,  $\eta$  is the quantum efficiency of the high speed photodetector,  $h\nu$  is the lasing photon energy, and  $P_0$  is the output power emitted from each facet and can be calculated from:

$$P_0 = \frac{1}{2} h\nu v_g \alpha_m P \quad \dots(11)$$

The nonlinear feedback can be performed by placing a nonlinear function in the electronic feedback loop from the PD back to the laser diode yielding a feedback current  $I_F(t - \tau) = f(I_P(t - \tau))$  with  $I_P$  the current produced from photodetector that is proportional to the laser output power. The nonlinear tent function is used as shown in Fig.2 [ 9 ].

### III-Simulation Results:

The chaotic VCSEL is illustrated in Fig.1. The straight linear and nonlinear feedback is considered. The output power emitted from VCSEL is received and converted to a current by a high speed photodetector as in equation (10 ). The feedback current added to the bias current, resulting in a delayed positive feedback.

In this investigation of the delayed differential equations (1) and (2) which describe the dynamics of the optoelectronic feedback laser, it's found that a chaotic waveform can be observed as the delay time and feedback strength is varied. The different parameter values of a VCSEL and high photodetector used in the simulation are listed in Table I.

There are two parameters  $\tau$  and  $\xi$  that can be varied. In this investigation one of these two parameters is held constant while the other parameter is varied. The numerical observation results generated by setting  $\tau$  at  $7.1 \times 10^{-9}$  sec. and varying  $\xi$ . Figures (3 - 5) show the time trace of the output power of VCSEL with optoelectronic linear feedback and corresponding power spectra. From the power spectra, it is clear that in Fig. 3a and 3b there is a fundamental frequency at  $f_1 = 1.7$  GHz and a second frequency at  $f_2 = 2.6$  GHz. The system in a two frequency as quasi - periodic oscillating state. An increase of the feedback strength  $\xi$  leads destabilization of the limit cycle and another two frequencies are created. The small ripples in the spectrum indicated that there is small instability in the laser.

The results in Fig.4 depict that the spectrum is broadened as  $\xi$  increases and the system enters the chaotic regime as in Fig.5b. The development of a broadband, a characteristic of chaos, is much higher in Fig.5b than in Fig.5a.

A time series and a corresponding nonlinear spectrum is shown in Fig.6. The complexity which arises with the nonlinear tent function is appearing and it produces high chaotic oscillations.

#### IV-Conclusion:

The chaotic oscillation of a VCSEL with optoelectronic feedback has been demonstrated. The optoelectronic systems based on fast chaotic dynamics has been proposed as possible alternative to classical encryption techniques relying on numerical algorithms [10].

The idea of introducing a nonlinear function that can be implemented electrically into feedback loop has been studied and demonstrated that it produces high chaotic oscillations. The feedback time - delayed current  $I_F$  add to the laser bias current to complete feedback loop.

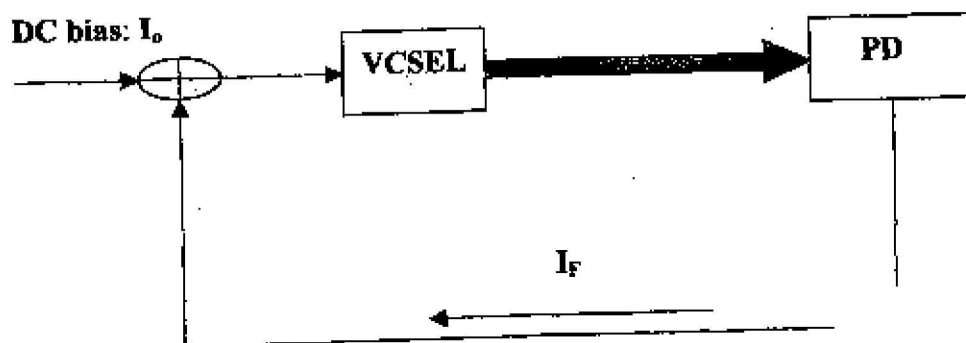
The numerical simulation has been achieved by MATLAB software package. The MATLAB software package integrates some high performance facilities in order to solve the scientific problem. The numerical problems are formulated in common programming languages then translated to S-function program. The work includes spectral analysis and time series of the output power of laser. For different values of  $\xi$ , Figures (3 - 5) represent the chaotic dynamics assuming  $I_F(t-\tau)$  to be proportional to the light output power. With the nonlinear feedback the chaotic dynamics larger than linear feedback for the same  $\xi$ .

#### References:

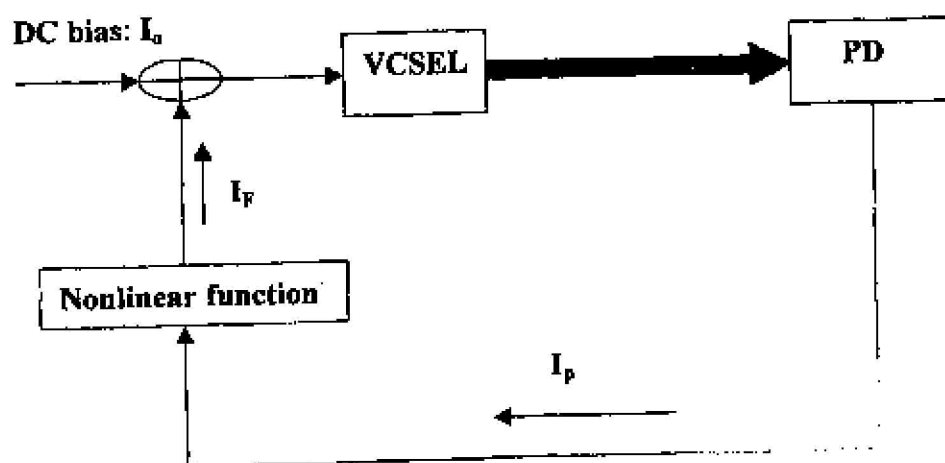
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Table 1: Parameters set in the numerical simulations.

Parameter	Value
$R_T$	0.95
$R_F$	0.95
$\alpha_{ex}$	$10 \text{ cm}^{-1}$
$\alpha_{ac}$	$10 \text{ cm}^{-1}$
$v_g$	$7.5 * 10^9 \text{ cm} \cdot \text{sec}^{-1}$
$n_0$	$10^{18} \text{ cm}^{-3}$
$\mu_g$	3:2
$\mu$	3.2
$\Delta\lambda_{sp}$	60 nm
$A_n$	$10^9 \text{ sec}^{-1}$
B	$10^{10} \text{ cm}^3 \cdot \text{sec}^{-1}$
$\lambda$	1.3 $\mu\text{m}$
$a_0$	$2.5 * 10^{16} \text{ cm}^2$
$\eta$	0.7
q	$1.6 * 10^{-19} \text{ C}$ .
$h\nu$	$1.52 * 10^{-19} \text{ J}$ .
$I_0$	8.1 mA.
C	$3 * 10^{-29} \text{ cm}^6 \cdot \text{sec}^{-1}$
D	5 $\mu\text{m}$
d	3 $\mu\text{m}$
L	8 $\mu\text{m}$



a



b

Figure 1: Schematic of chaotic semiconductor laser with delayed optoelectronic feedback. (a) linear feedback and (b) nonlinear feedback. VCSEL: laser source; PD: photodetector;  $I_0$ : dc bias current;  $I_f$ : feedback current;  $I_p$ : photodetector current.

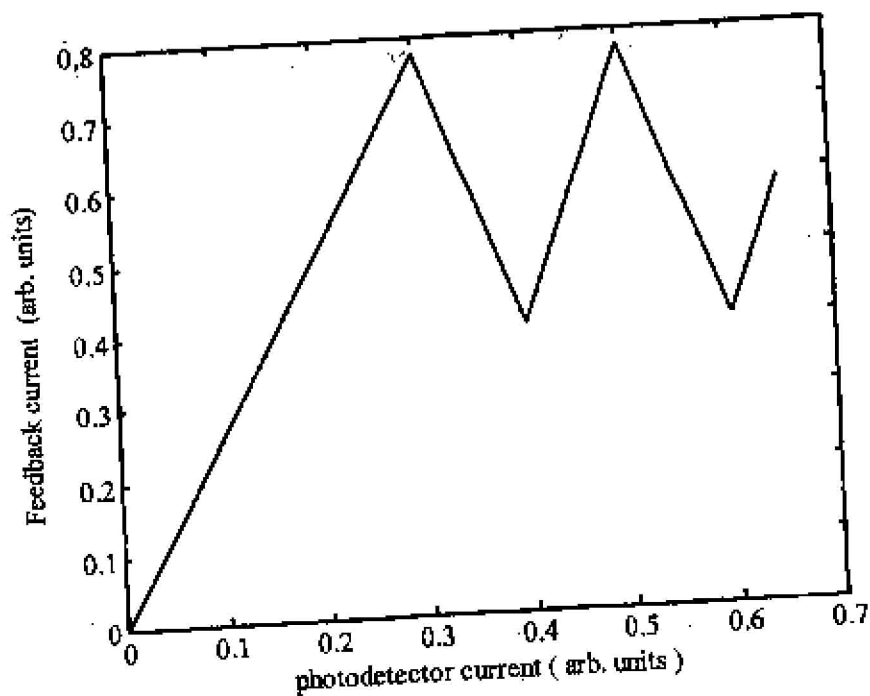


Figure 2: The nonlinear tent function, used as the nonlinear element in feedback loop

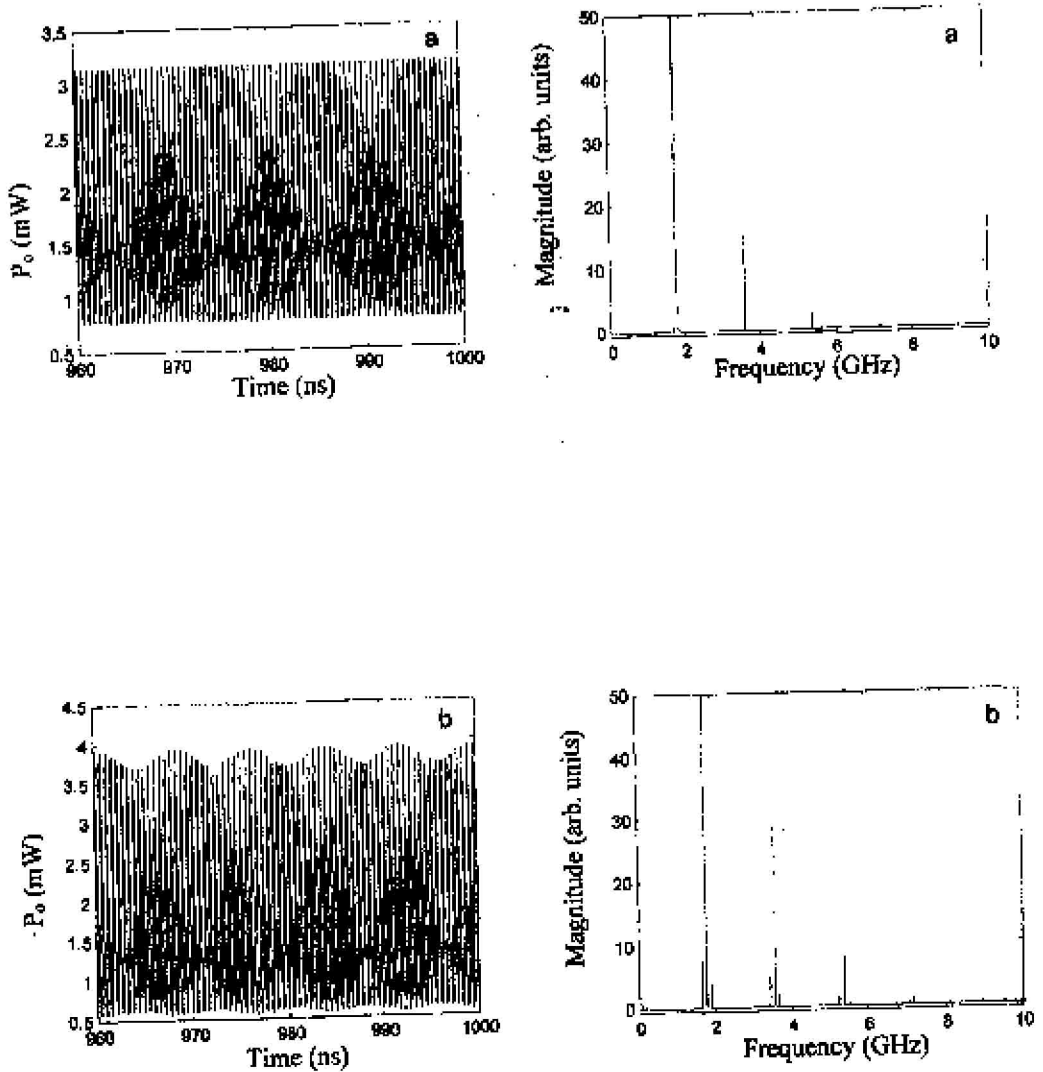


Figure 3: Time series ( first column ) and corresponding power spectra ( second column ) at different feedback strength: ( a )  $\xi = 0.25$  and ( b )  $\xi = 0.27$ .



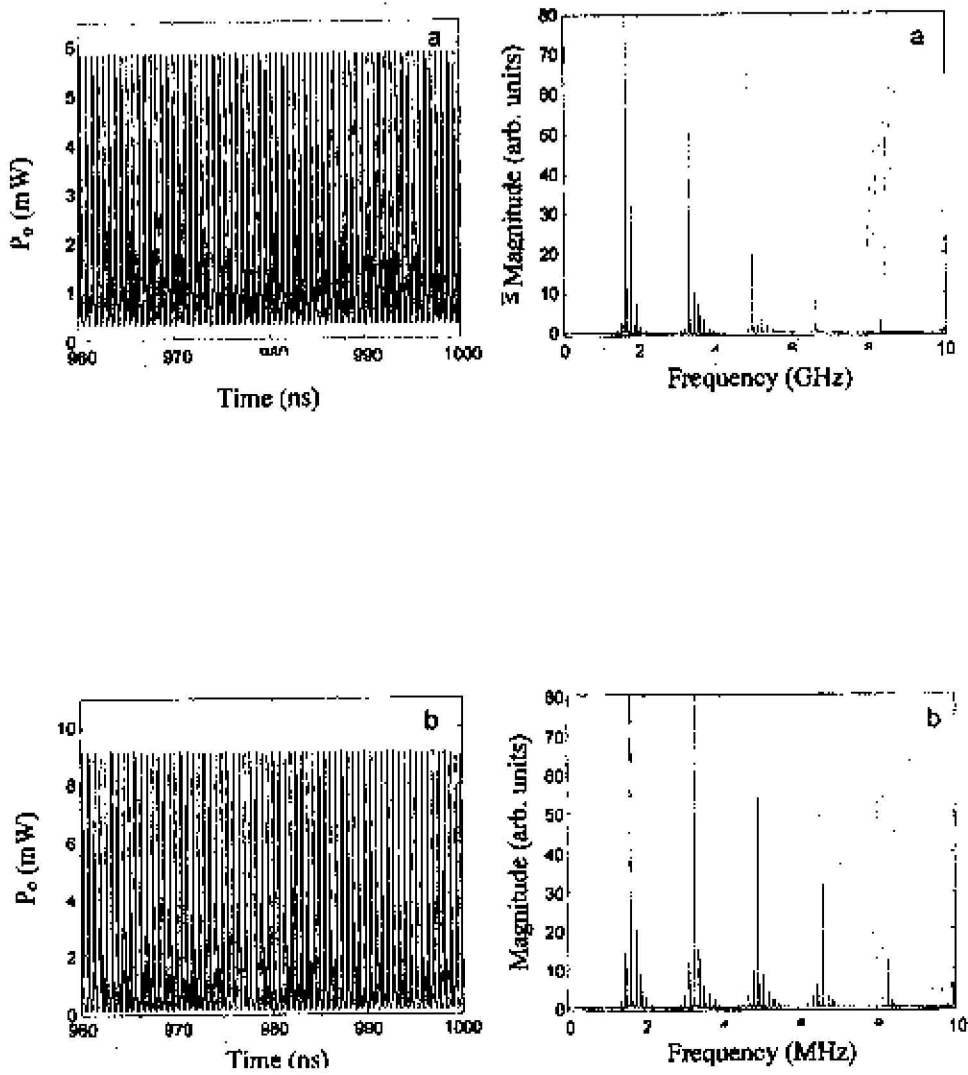


Figure 4: Time series ( first column ) and corresponding power spectra ( second column ) at different feedback strength: ( a )  $\xi = 0.3$  and ( b )  $\xi = 0.4$ .

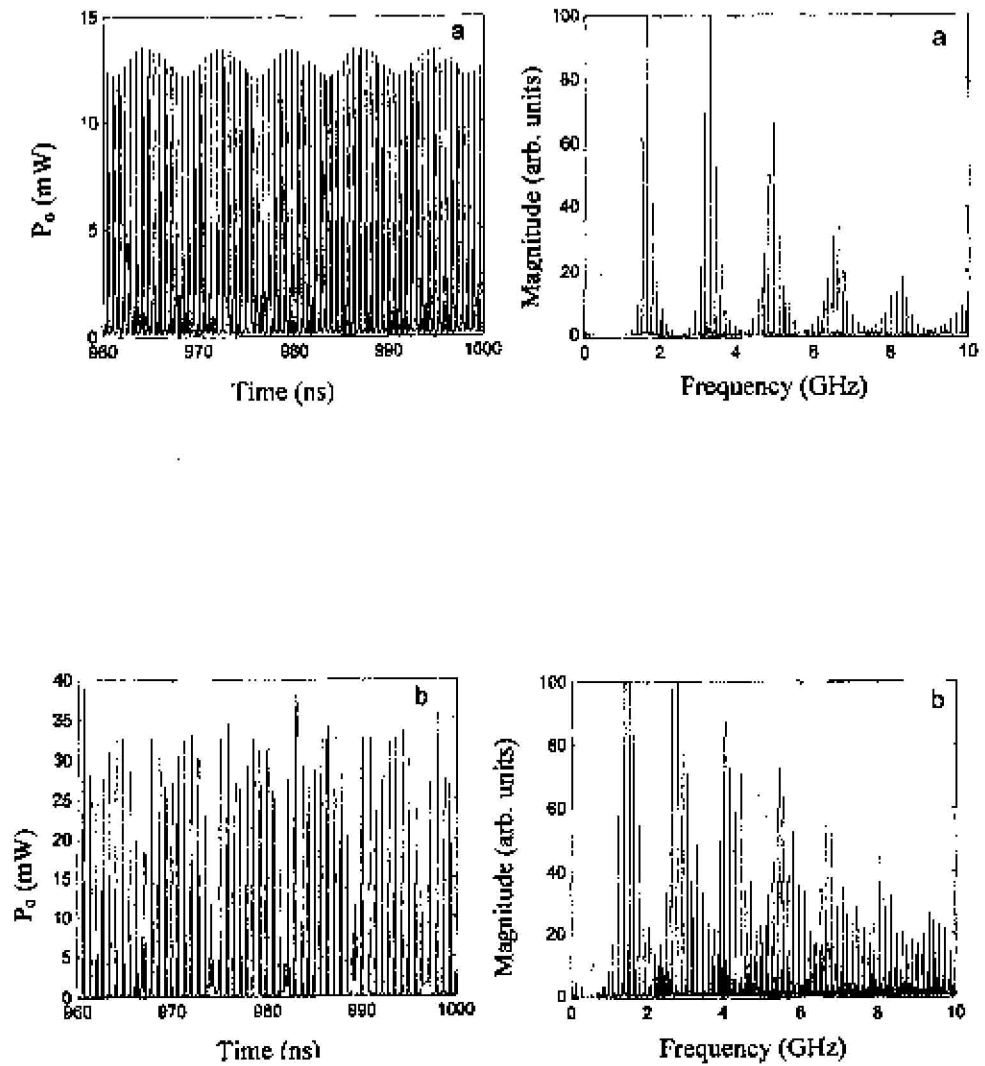


Figure 5: Time series (first column) and corresponding power spectra (second column) at different feedback strength: (a)  $\xi = 0.5$  and (b)  $\xi = 0.65$ .

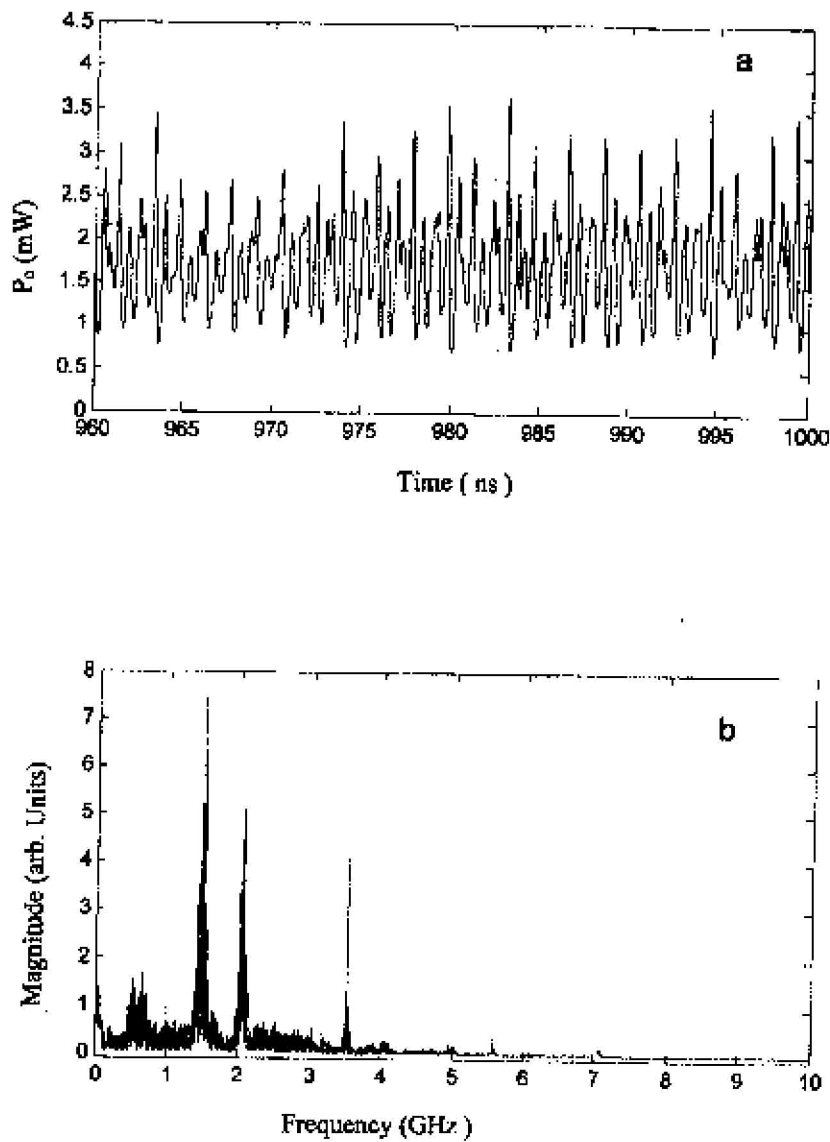


Figure 6: ( a ) Time series and ( b ) corresponding power spectra from chaotic optoelectronic nonlinear feedback VCSEL.