Abstract
In this paper, a new technique for multi-robot localization in an unknown environment, called the leader-follower localization algorithm is presented. The framework utilized here is one robot that goes about as a leader and different robots are considered as followers distributed randomly in the environment. Every robot equipped with RP lidar sensors to scan the environment and gather information about every robot. This information utilized by the leader to distinguish and confine every robot in the environment. The issue of not noticeable robots is solved by contrasting their distances with the leader. Moreover, the equivalent distance robot issue is unraveled by utilizing the permutation algorithm. Several simulation scenarios with different positions and orientations are implemented on (3-7) robots to show the performance of the introduced technique.

Index Terms— RP Lidar sensor, Multi-robot system, Localization, Leader follower algorithm.

I. INTRODUCTION
Multi-robot system has a high potential to solve different certifiable issues, for example, investigation, building up system inclusion, search and salvages missions, also used in dangerous places such as fires and explosive fields [1-4]. At the point when multi robots are working together on explicit tasks, their areas require precise intending to guarantee that the robots are not conflicting with one another [5]. Extraordinary consideration is given to structure effective calculations for localization. The term localization implies that to discover the pose (position and orientation) of robots in an environment according to the information obtained from sensors equipped with each one [6-8]. Localization can be grouped into two kinds: absolute localization, in which every robot tries to decide its pose as for some global coordinate system, and relative localization, in which every robot tries to decide the pose of each other robots in the group, with respect to itself [9-10]. As indicated by the design utilized for robots situating, the localization can be isolated into centralized and decentralized [11-13]. In the centralized scheme, There is a central control robot that has the global data about the environment just as all data about the robots [6], The upside of the centralized scheme is that increasingly precisely on account of location estimation and needs less time to correct an error. Nevertheless, in this scheme, if the central control robot fails another robot must be accessible or else the entire system disabled [14]. While in the decentralized scheme, each root of the multi-robot system processes its information locally. The upside of a decentralized scheme is that it can all the more likely react to obscure or evolving situations, and more often than not has better dependability, adaptability, versatility, and vigor. In any case, the arrangements they reach are frequently imperfect [15].

In this paper, we introduce a leader-follower localization algorithm that controls the pose of each robot in the multi-robot system. The approach aims to make the leader robot identify robots in the formation and localize them with the help of the information obtained from the RP lidar sensor equipped with each robot. The use of the RP lidar sensor due to its qualities of a full 360 degree of scanning, low cost and can take
large number of distances reading per second [16-19]. The details of these approaches will be discussed in section 2, section 3 displays the simulation results and the conclusion will be in section 4.

II. LEADER FOLLOWER LOCALIZATION ALGORITHM

In this section, the leader-follower localization algorithm is introduced as a new algorithm for a multi-robot localization system. This system consists of four follower robots to scan the environment and one leader robot to identify and localize the other robots. Each follower robot is equipped with RP lidar sensor to scan the environment and send the scan information to the leader. The implementation process of this algorithm is performed according to the following subsections.

A. Robots identification

The leader-follower localization system is designed to identify each robot in the environment using only the distances among these robots. Each follower robot in the system sends its scanned information to the leader one. The leader robot identifies each robot in the environment by comparing and matching the collected information from these follower robots. The process of identification is implemented according to the following steps:

1. Each follower robot scans the environment by rotating its RP Lidar laser sensor 360 degrees to measure the distances between it and each other robot. The scan process starts from it is the current direction and rotates step by step in an anti-clockwise direction until complete the 360 degrees as shown in Fig.1. The collected information of each follower robot store in a one-dimensional array represents the distances between it and the other robots in the environment.

$$\begin{align*}
R_1 &= [L_{14}, L_{12}, L_{10}, L_{13}] \\
R_2 &= [L_{23}, L_{20}, L_{21}, L_{24}] \\
R_3 &= [L_{31}, L_{34}, L_{30}, L_{32}] \\
R_4 &= [L_{40}, L_{43}, L_{41}, L_{42}] 
\end{align*}$$

Where R1, R2, R3, and R4 are the follower robots 1, 2, 3, and 4 respectively. Each follower robot sends its scanned information to the leader robot.

2. The leader starts to rearrange the elements of the array $A$ according to robot locations in array $B$. The rearranging process starts by comparing the distances in row 1 and row 2 of the array $A$. The matching values represent
the distance between follower robots 1 and 2 (L_{12} and L_{21} in rows 1 and 2). These values are placed in the second column in row 1 and the first column in row 2 as shown in array C. The second comparison is between the first row and the third row. The matching values represent the distance between follower robots 1 and 3, (L_{13} and L_{31} in rows 1 and 2). These values are placed in the third column in row 1 and second column in row 3 as shown in array C. These comparisons are repeated until complete all the elements of the array C. The diagonal elements of this array represent the distances between the leader and the follower robots (L_{10}, L_{20}, L_{30}, and L_{40}).

\[
B = \begin{bmatrix}
R_0 & R_1 & R_2 & R_3 \\
R_1 & R_0 & R_3 & R_4 \\
R_1 & R_2 & R_3 & R_0 \\
R_1 & R_2 & R_3 & R_0
\end{bmatrix}
\] (6)

Where \( R_0 \) is the leader robot.

\[
C = \begin{bmatrix}
L_{10} & L_{12} & L_{13} & L_{14} \\
L_{21} & L_{20} & L_{23} & L_{24} \\
L_{31} & L_{32} & L_{30} & L_{34} \\
L_{41} & L_{42} & L_{43} & L_{40}
\end{bmatrix}
\] (7)

3. The final step in the identification process is represented by constriction four one dimensional arrays, each one is sent to the corresponding follower robot. The elements of these arrays are obtained from the matching between the elements of the received four one dimensional arrays which contain the array C. These four arrays are sent to the corresponding follower robots.

\[
R_1 = [R_1, R_2, R_0, R_3] \\
R_2 = [R_3, R_0, R_1, R_4] \\
R_3 = [R_1, R_4, R_0, R_2] \\
R_4 = [R_6, R_3, R_1, R_2]
\] (8-11)

### B. Robot localization

The localization process between any two robots depends on the reading of the laser beam. Each laser beam intersects any robot in more than one point. The localization process is accomplished by choosing the suitable direction and distance between the two robots which achieved according to the following steps:

1. Test the distance between the laser beam and the intersected robot to be less than the radius of that robot. Fig. 2 shows the method used for estimating the distance \( h \) according to the following equation.

\[
a = (x_2 - x_1)/(y_2 - y_1) \\
b = -1 \\
c = y_1 - a * x_1 \\
h = abs((a * x_3 + b * y_3 + c)/sqrt(a^2 + b^2))
\] (12-15)

![Fig. 2. Measuring the distance between the robot and the intersected laser beam.](image)

2. Each laser beam intersects any robot in more than one point as shown in Fig. 3. Since the robot has a circular shape so that the laser beam with the shortest distance gives a more accurate direction of the robot position. \( d_2 \) represents the shortest distance.
3. The robot location depends on the chosen laser beam which distance and orientation are known. From Fig. 4, the laser beam orientation has $\delta$ degree error from the correct orientation of the direct path between robots 1 and 3. Also, it has $(L_{13} - d)$ distance error between the two robots. Where $d$ is the actual distance to the other robot.

$$x_3 = x_1 + (d + r) \cdot \cos(\theta + \delta)$$
$$y_3 = y_1 + (d + r) \cdot \sin(\theta + \delta) \quad (16)$$

Eq. 16 compute the actual coordinates to the robot 3 and Eq. 17 compute the estimate coordinate axis to the robot.

Where $r$ is the robot radius.

$$x_{es} = x_1 + (L_{13} + r) \cdot \cos(\theta)$$
$$y_{es} = y_1 + (L_{13} + r) \cdot \sin(\theta) \quad (17)$$

4. The distance between the actual and the estimated robot locations represent the error $err$.

$$err = ((x_3 - x_{es})^2 + (y_3 - y_{es})^2)^{0.5} \quad (18)$$

C. The invisible robot problem

The invisible robot problem occurs when three of the robots are located on a straight line as shown in Fig. 5 (robots 1, 4 and 2). Both the laser beams of the robots 1 and 2 cannot detect each other because the robot 4 lies on the straight line between them. The solving of this problem is according to the following steps:

1. As in step 1 of the identification subsection, each follower robot scans the environment to measure the distance between it and each other robot. The collected information of each follower robot store in a one-dimensional array represents the distances between it and the other robots in the environment. Eqs. 19, 20, 21 and 22 represent the arrays of distances that collected by the follower robots that are shown in Fig. 5.

$$d_i = \text{distance between robot } i \text{ and robot } 0$$
$$d_{i1} = \text{distance between robot } i \text{ and robot } 1$$
$$d_{i2} = \text{distance between robot } i \text{ and robot } 2$$
$$d_{i3} = \text{distance between robot } i \text{ and robot } 3$$

Eq. 19 compute the actual coordinates to the robot 3 and Eq. 17 compute the estimate coordinate axis to the robot.
element values $x$ in the arrays means the distance between the two robots is unknown. Each follower robot sends its scanned information to the leader robot.

$$R_1 = [L_{14}, L_{10}, L_{13}, x]$$  \hspace{1cm} (19)

$$R_2 = [L_{23}, L_{20}, L_{24}, x]$$  \hspace{1cm} (20)

$$R_3 = [L_{31}, L_{34}, L_{30}, L_{32}]$$  \hspace{1cm} (21)

$$R_4 = [L_{42}, L_{40}, L_{43}, L_{41}]$$  \hspace{1cm} (22)

2. The leader robot collects the received distance information in a single two-dimensional array $D$ (eq. 23). The leader starts to rearrange the elements of the array $D$ according to robot locations in array $B$.

$$D = \begin{bmatrix}
L_{14} & L_{10} & L_{13} & x \\
L_{23} & L_{20} & L_{24} & x \\
L_{31} & L_{34} & L_{30} & L_{32} \\
L_{42} & L_{40} & L_{43} & L_{41}
\end{bmatrix}$$  \hspace{1cm} (23)

3. The rearranging process starts by comparing the distances in row 1 with rows 2, 3 and 4 of the array $D$. The matching values mean the distance between any two robots is equal and this value is placed in new array $E$. These comparisons are repeated until complete all the array $E$ as in array $C$ in the robot identification subsection. The difference between array $C$ and array $E$ is the unknown values $x$. The unknown values in array $D$ are located on the first and the second rows (rows with 3 values only). As shown in Fig. 5 there are three paths pass from robot 1 to robot 2. The shortest path represents a straight line between robots 1 and 2 (the distance $L_{12}$ and $L_{21}$). To find these distances we used the permutation algorithm among all the distances in rows 1 and 2 in array $D$ ($(L_{14}, L_{23})$, $(L_{14}, L_{20})$, $(L_{14}, L_{24})$, $(L_{10}, L_{23})$, $(L_{10}, L_{20})$, $(L_{10}, L_{24})$, $(L_{13}, L_{23})$, $(L_{13}, L_{20})$ and $(L_{13}, L_{20})$). According to the permutation algorithm, the sum of any two values of the two rows with a minimum value represents the straight distance between the robots 1 and 2 (the distance $L_{12}$ and $L_{21}$). The diagonal elements of this array represent the distances between the leader and the follower robots ($L_{10}$, $L_{20}$, $L_{30}$, and $L_{40}$).

$$E = \begin{bmatrix}
L_{10} & x & L_{13} & L_{14} \\
x & L_{20} & L_{23} & L_{24} \\
L_{31} & L_{32} & L_{30} & L_{34} \\
L_{41} & L_{42} & L_{43} & L_{40}
\end{bmatrix}$$  \hspace{1cm} (24)

4. The final step in the identification process is similar to the final step in the robot identification subsection. Four one dimensional arrays (Eq. 25, 26, 27 and 28) must be constructed and each one sends to the corresponding follower robot. These four arrays are sent to the corresponding follower robots.

$$R_1 = [R_4, R_0, R_3, R_2]$$  \hspace{1cm} (25)

$$R_2 = [R_3, R_0, R_4, R_1]$$  \hspace{1cm} (26)

$$R_3 = [R_1, R_4, R_0, R_2]$$  \hspace{1cm} (27)

$$R_4 = [R_2, R_0, R_3, R_1]$$  \hspace{1cm} (28)

**D. The equal distance problem**

The equal distance problem occurs when the laser beam of one of the follower robots measures the same distances from two of the other robots as shown in Fig. 6 (Robot 4 has two equal distances to the robots 1 and 2). This makes a problem for the leader in the construction of the two-dimensional array. The solving of this problem is according to the following steps:

1. Use the same procedure in step 1 in the identification subsection, where each follower robot scans the environment to measure the distance between it and each
other robot. The collected information of each follower robot (Fig. 6) Store in a one-dimensional array as shown in Eq. 29, 30, 31 and 32. Each follower robot sends it is scanned information to the leader robot.

\[
\begin{align*}
R_1 &= [L_{14}, L_{12}, L_{10}, L_{13}] \\
R_2 &= [L_{23}, L_{20}, L_{21}, L_{24}] \\
R_3 &= [L_{31}, L_{34}, L_{30}, L_{32}] \\
R_4 &= [L_{42}, L_{40}, L_{43}, L_{41}] 
\end{align*}
\]

(29) (30) (31) (32)

Fig. 6. The equal distance problem.

2. The information received from the leader robot store in a single two-dimensional array \( F \) (eq. 33). The equal distance problem occurs in row 4 (\( L_{42} \) and \( L_{41} \)) in array \( F \).

\[
F = \begin{bmatrix}
L_{14} & L_{12} & L_{10} & L_{13} \\
L_{23} & L_{20} & L_{21} & L_{24} \\
L_{31} & L_{34} & L_{30} & L_{32} \\
L_{42} & L_{40} & L_{43} & L_{41}
\end{bmatrix}
\]

(33)

3. The leader starts to rearrange the elements of the array \( F \) according to robot locations in array \( B \). The rearranging process has no problem in row 1, 2 and 3 but the problem occurs in row 4 (\( L_{42} \) and \( L_{41} \)). These comparisons are repeated until complete all the array \( G \). The diagonal elements of this array represent the distances between the leader and the follower robots (\( L_{10}, L_{20}, L_{30} \) and \( L_{40} \)).

\[
G = \begin{bmatrix}
L_{10} & L_{12} & L_{13} & L_{14} \\
L_{21} & L_{20} & L_{23} & L_{24} \\
L_{31} & L_{32} & L_{30} & L_{34} \\
L & L & L_{43} & L_{40}
\end{bmatrix}
\]

(34)

4. Four one dimensional arrays (Eq. 35, 36, 37 and 38) must be constructed and each one sends to the corresponding follower robots.

\[
\begin{align*}
R_1 &= [R_4, R_2, R_0, R_3] \\
R_2 &= [R_1, R_0, R_3, R_4] \\
R_3 &= [R_1, R_2, R_0, R_4] \\
R_4 &= [R, R, R_3, R_0]
\end{align*}
\]

(35) (36) (37) (38)

5. The equal distance problem occurs in robot 4. It has two possible distributions for the robots: \( (R_2, R_1, R_3, R_0) \) or \( (R_1, R_2, R_3, R_0) \). The following procedure is used to solve this problem:

a- Assume the one dimensional array \( (R_2, R_1, R_3, R_0) \) is true.

b- Compute the coordinate axis \( (x_2, y_2) \) for the follower robot \( R_2 \).

\[
\begin{align*}
x_2 &= x_4 + L_{42} \cdot \cos(\theta) \\
y_2 &= y_4 + L_{42} \cdot \sin(\theta)
\end{align*}
\]

(39)

c- Compute the distance \( L \) between the follower robot \( R_2 \) and the leader robot \( R_0 \).

\[
L = ((x_2 - x_0)^2 + (y_2 - y_0)^2)^{0.5}
\]

(40)

d- If the computed distance \( L \) equal to the distance \( L_{20} \) then the first one dimensional array \( (R_2, R_1, R_3, R_0) \) is true, else the second one-dimensional array \( (R_1, R_2, R_3, R_0) \) is true as shown in Fig. 7.
III. SIMULATION RESULTS

In this paper, a new algorithm for robot localization system in a local knowledge environment is simulated using visual Basic program. The simulations are implemented in an environment that has a different number of robots one works as a leader and the others work as followers as shown in Fig. 8. Each follower robot is equipped with RP Lidar laser sensor to scan the environment and to measure the distances with respect to the other robots. The simulation is repeated 50 times on a single leader robot with different numbers of follower robots (from 3 to 7 robots) which distributed randomly in (500 pixels * 500-pixel environment).

The parameters used in these simulations are:
1- Distance between the leader and the follower robots.
2- Rotating angle of the RP Lidar laser sensor.

Fig. 9, 10, 11 and 12 are used to show the number of the reading samples for each laser beam with respect to the rotating angle of this beam (1, 3 and 5 degrees) and the distances between the leader and the follower robot (50, 100, 150 and 200 pixels).
rotating is 5 degrees and 12 reading when the angle is 3 degrees and 23 reading when the angle is 1 degree.

The second simulation result (Fig. 14) is implemented in a follower robot far 100 pixels from the leader and repeated for 1, 3 and 5 degrees rotating angle for the laser beam. The number of laser reading is 5 when the angle of rotating is 5 degrees and 11 reading when the angle is 3 degrees and 23 reading when the angle is 1 degree.

The third simulation result (Fig. 15) is implemented in a follower robot far 150 pixels from the leader and repeated for 1, 3 and 5 degrees rotating angle for the laser beam. The number of laser reading is 4 when the angle of rotating is 5 degrees and 11 reading when the angle is 3 degrees and 22 reading when the angle is 1 degree.

The fourth simulation result (Fig. 16) is implemented in a follower robot far 200 pixels from the leader and repeated for 1, 3 and 5 degrees rotating angle for the laser beam. The number of laser reading is 4 when the angle of rotating is 5 degrees and 10 reading when the angle is 3 degrees and 21 reading when the angle is 1 degree.

Fig. 11. The simulation, distance 150 pixels between the leader and the followers.

Fig. 12. The simulation, distance 200 pixels between the leader and the followers.

Fig. 13. The simulation results for 50 pixels distance between the leader and the follower robots.
Fig. 14. The simulation results for 100 pixels distance between the leader and the follower robots.

Fig. 15. The simulation results for 150 pixels distance between the leader and the follower robots.
The above simulation results show that as the distance between the leader and the follower robots increase the number of laser beam reading decrease and this lead to decrease in the accuracy of estimation the robot location. Also, as the rotating angle decreases the number of the laser beam reading also decrease. In this case the accuracy of robot localization also decreases.

Figs. 17, 18 and 19 show the next group of simulation which implemented on different number of follower robots with different distances and repeated for different angle of rotation of the laser beam (1, 3 and 5 degrees).
The simulation results (Fig. 20) show that the accuracy of reading increases as the distance between the followers and leader robots decreases. From Fig.20, it is clear that the nearest follower robot has the highest accuracy reading. Also, this figure studies the effect of the rotation angle step on the accuracy of the laser sensor readings. As the rotating angle of the laser beam decreases the accuracy of readership increases.

IV. CONCLUSIONS

In this paper, an efficient algorithm was proposed to identify and localize multi-robot in the leader-follower environment. Each follower robot equipped with RP Lidar sensor to scan the environment and compute the distance to other robots. The simulation is implemented on the environment with the different number of follower robots (3 to 7 robots) and different distances from the leader robot (50, 100, 150 and 200 pixels) and also, different rotating angle to the laser sensor (1, 3 and 5 degrees). The results show that the suggested algorithm has a better performance in identification and localization multi-robot systems. The simulation results show that as the distance between the leader and the follower robots decreases the accuracy is increased, and as the rotating angle of the laser sensor decreases the accuracy of the localization increases.

REFERENCES


